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Übungen zu

Computational Finance II

Exercise 1 Discrete Dividend Payment

Assume that a stock pays a dividend D at ex-dividend date t_D, with $0 < t_D < T$.

a) Assume that a known dividend is paid once per year. Calculate a corresponding continuous dividend rate δ under the assumptions

$$\dot{S} = (\mu - \delta)S\,,\ \mu = 0, \quad S(1) = S(0) - D > 0\,.$$

Generalize the result to general growth rates μ and arbitrary t_D. (To apply for options, note that this assumes $T = 1$.)

b) Define for an American put with strike K

$$\tilde{t} := t_D - \frac{1}{r} \log\left(\frac{D}{K} + 1\right)\,.$$

Assume $S = 0$, $r > 0$, $D > 0$, and a time instant t in $\tilde{t} < t < t_D$. Argue that instead of exercising early it is reasonable to wait for the dividend.
Note: For $\tilde{t} > 0$, depending on S, early exercise may be reasonable for $0 \le t < \tilde{t}$.

c) In Section 1.1 let N_i denote the number of nodes of the standard (non-recombining) binomial S-tree at t_i, and let t_k be the ex-dividend date.
Show $N_{k+i} = (i+1)(k+1)$ for $i > 0$.

Exercise 2 Programming Project

Program and test the Algorithm of Section 1.1 (binomial tree in case of a discrete dividend). Parameters: $T = 0.5$, $S_0 = 50$, $K = 55$, $r = 0.1$, $\sigma = 0.4$, $M = 10$ time steps, $D = 5$, $t_D = 0.31$. Plot the S-tree, and approximate $V(S_0, t)$ for an American put.

Exercise 3 Changing a Function

Assume a function Ψ, for example the payoff of a vanilla call $\Psi(S) := (S - K)^+$. Ψ can be approximated by $\overline{\Psi}$,

$$\overline{\Psi}(S) := \frac{1}{2\xi} \int_{-\xi}^{\xi} \Psi(S - y)\,\mathrm{d}y\,,$$

for a suitable chosen small $\xi > 0$.

a) What is the "advantage" of $\overline{\Psi}$ compared to Ψ?

b) Calculate $\overline{\Psi}$ analytically for the payoff of a vanilla call and of a digital call.

c) Set up an algorithm to calculate $\overline{\Psi}$ numerically for a given function Ψ. Use trapezoidal quadrature and program it on a computer.

Literatur zur Lehrveranstaltung

Skript zu den Grundlagen: www.compfin.de

Topics in CF (Illustrationen und Ergänzungen): www.compfin.de

Lehrbuch: R. Seydel: *Tools for Computational Finance*. 5. Auflage. Springer (2012)