Übungen zu

Computational Finance II

Exercise 5 Calculate the Standard Normal Distribution Function

There are several ways to approximate the integral

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\frac{t^2}{2})\, \mathrm{d}t\,.$$

Here we implement two methods.

a) Construct an algorithm to calculate the *error function*

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-t^2)\, \mathrm{d}t$$

and use $\operatorname{erf}(x)$ to calculate $F(x)$.
Use as quadrature method the composite trapezoidal rule with n steps to obtain an approximation $\tilde{F}_n(x)$. Improve this using extrapolation ("Romberg's quadrature").

b) Define

$$z := \frac{1}{1 + 0.2316419x}$$

and the coefficients

$$\begin{aligned} a_1 &= 0.319381530 & a_4 &= -1.821255978 \\ a_2 &= -0.356563782 & a_5 &= 1.330274429 \\ a_3 &= 1.781477937 \end{aligned}$$

Then for $x > 0$

$$\tilde{F}(x) := 1 - f(x)z((((a_5 z + a_4)z + a_3)z + a_2)z + a_1)$$

approximates $F(x)$ with an absolute error not exceeding 10^{-7}. For $x < 0$ apply $F(x) = 1 - F(-x)$.
Count the number of arithmetic operations, and implement $\tilde{F}$.

c) Full accuracy for comparison can be obtained by the generic code `derf`. For a series of values x evaluate the errors of the above algorithms. Enter the computing times and the errors of $\tilde{F}$ and F_n for several n into a double-logarithmic diagram depicting computing time over relative accuracy.

Exercise 6 Extrapolation

Assume a (differential) equation, and let $\eta^* \in \mathbb{R}$ represent its exact solution. For a discretization, Δ denotes the grid size of a numerical approximation scheme, and $\eta(\Delta)$ the corresponding approximating solution. Further assume an error model

$$\eta(\Delta) - \eta^* = c\,\Delta^q,$$

with $c, q \in \mathbb{R}$. q is the *order* of the approximation scheme. Suppose that for two grid sizes Δ_1, Δ_2 with

$$\Delta_2 = \frac{1}{2}\Delta_1$$

approximations $\eta_1 := \eta(\Delta_1)$, $\eta_2 := \eta(\Delta_2)$ are calculated.

a) For the case of a known η^* (or η^* approximated with very high accuracy) establish a formula for the order q out of η^*, η_1, η_2.

b) For a known order q show that

$$\eta^* = \frac{1}{2^q - 1}\left(2^q \eta_2 - \eta_1\right).$$

In general, the error model holds only approximately. Hence this formula for η^* is only an approximation to the exact η^* (“extrapolation”).