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Übungen zu

Computational Finance II

Exercise 7 Variants of the Binomial Method

a) Use the equation $p = 1/2$ (instead of $ud = 1$) to show

$$u = \mathrm{e}^{r\Delta t}\,(1 + \sqrt{\mathrm{e}^{\sigma^2\Delta t} - 1})$$
$$d = \mathrm{e}^{r\Delta t}\,(1 - \sqrt{\mathrm{e}^{\sigma^2\Delta t} - 1})\,.$$

b) Price Evolution for the Binomial Method:
For $\beta := \frac{1}{2}(\mathrm{e}^{-r\Delta t} + \mathrm{e}^{(r+\sigma^2)\Delta t})$ and $u = \beta + \sqrt{\beta^2 - 1}$ show

$$u = \exp\left(\sigma\sqrt{\Delta t}\right) + O\left(\sqrt{(\Delta t)^3}\,\right)\,.$$

c) For the CRR choice

$$u := \mathrm{e}^{\sigma\sqrt{\Delta t}},\ d := \mathrm{e}^{-\sigma\sqrt{\Delta t}},\ \tilde{p} := \frac{1}{2}\left(1 + \frac{r - \sigma^2/2}{\sigma}\sqrt{\Delta t}\right)$$

show that $\tilde{p}$ is a first-order approximation of p.

Exercise 8 Trinomial Model

Extend the classical binomial model to a trinomial model as follows: Allow for three prices S_{i+1} of the underlying at t_{i+1}, namely,

$$\begin{array}{ll} uS_i & \text{with probability } p_1 \\ mS_i & \text{with probability } p_2 \\ dS_i & \text{with probability } p_3\,. \end{array}$$

For the six parameters u, m, d, p_1, p_2, p_3 six equations are needed. Clearly, the probabilities must be nonnegative, and $p_1 + p_2 + p_3 = 1$ must hold.

a) Set up the two equations that equate expectation and variance with the corresponding values of the continuous model (similar as for the binomial model).

b) The tree should be recombining. Cast this requirement into an equation.

c) For the special choice of equal probabilities derive the parameters.

Hint: For

$$\alpha := e^{r\Delta t}\,, \quad \beta := e^{\sigma^2\Delta t}$$

show

$$m = \frac{\alpha}{2}(3 - \beta)\,, \quad u = \rho + \sqrt{\rho^2 - m^2} \quad \text{for } \rho := \frac{\alpha}{4}(\beta + 3)$$

d) How to avoid cancellation in the evaluation of u?

e) How many arithmetic operations are needed for the trinomial method with $\Delta t = T/M$? (without u, m, d)

f) Compare the efficiency of binomial approach with that of the trinomial approach.