Übungen zu

Computational Finance II

Exercise 9 Perpetual Put Option

For $T \to \infty$ it is sufficient to analyze the ODE

$$\frac{\sigma^2}{2} S^2 \frac{\mathrm{d}^2 V}{\mathrm{d}S^2} + (r-\delta) S \frac{\mathrm{d}V}{\mathrm{d}S} - rV = 0\,.$$

Consider an American put contacting the payoff $(K-S)^+$ at $S = S_\mathrm{f}$. Show:

a) Upon substituting the boundary condition for $S \to \infty$ one obtains

$$V(S) = c\left(\frac{S}{K}\right)^{\lambda_2},$$

where $\lambda_2 = \frac{1}{2}\left(1 - q_\delta - \sqrt{(q_\delta - 1)^2 + 4q}\right)$, $q = \frac{2r}{\sigma^2}$, $q_\delta = \frac{2(r-\delta)}{\sigma^2}$
and c is a positive constant.
Hint: Apply the transformation $S = Ke^x$. (The other root λ_1 drops out.)

b) V is convex.

For $S < S_\mathrm{f}$ the option is exercised; then its intrinsic value is $K - S$. For $S > S_\mathrm{f}$ the option is not exercised and has a value $V(S) > K - S$. The holder of the option decides when to exercise. This means, the holder makes a decision on the contact S_f such that the value of the option becomes maximal.

c) Show: $V'(S_\mathrm{f}) = -1$, if S_f maximizes the value of the option.
Hint: Determine the constant c such that $V(S)$ is continuous in the contact point.

Exercise 10 Semidiscretization, Method of Lines

For a semidiscretization of the Black–Scholes (BS) equation consider the semidiscretized domain

$$0 \le t \le T\,, \quad S = S_i := i\Delta S\,, \quad \Delta S := \frac{S_{\max}}{m}\,, \quad i = 0, 1, \ldots, m$$

for suitable values of $S_{\max} > K$ and m. On this set of lines parallel to the t-axis define for $\tau := T - t$ and $1 \le i \le m-1$ functions $w_i(\tau)$ as approximation to $V(S_i, \tau)$.

a) Using the standard second-order difference schemes, derive the ODE system $\dot{w} = Bw$ that approximates the BS equation (up to boundary conditions). Here w is the vector with components $w_1, \ldots, w_{m-1}$, and $\dot{w}$ denotes differentiation w.r.to τ. Show that B is a tridiagonal matrix, and calculate its coefficients.

b) For a European option assume Dirichlet boundary conditions for $w_0(\tau)$ and $w_m(\tau)$ and set up a vector c such that

$$\dot{w} = Bw + c \tag{$*$}$$

realizes the ODE system with correct boundary conditions, and with initial conditions taken from the payoff.

Recommendation

c) Use an implicit Euler scheme (or the BDF2 formula), and implement this scheme for the initial-value problem with $(*)$ and a European call option.

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Klausur: Mittwoch, 19.2.2014, 14-15 Uhr in H IV