Übungen zu

Computational Finance II

Background: Two versions of the same equation are

$$\begin{aligned}\frac{\partial V}{\partial t} &+ \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + (r-\delta_1) S_1 \frac{\partial V}{\partial S_1} - rV \\ &+ \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + (r-\delta_2) S_2 \frac{\partial V}{\partial S_2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} = 0\end{aligned} \tag{1}$$

and

$$-\nabla \cdot (D(x,y)\nabla u) + b(x,y)^{tr}\nabla u + ru = u_t = -\frac{\partial}{\partial \tau} u\,. \tag{2}$$

Exercise 11

a) Prove the equivalence of (1) and (2). Specialize this to the one-dimensional case of the Black–Scholes equation.

b) Show

$$b^{tr}\nabla u + ru = \nabla \cdot (bu) + \gamma u$$

and determine γ.

c) With the transformation

$$x := \log(\frac{S_1}{K_1}),\ \ y := \log(\frac{S_2}{K_2})$$

and writing $u(x,y,t)$ for V leads to the PDE

$$\begin{aligned}u_t &+ \tfrac{1}{2}\sigma_1^2 u_{xx} + (r - \delta_1 - \tfrac{1}{2}\sigma_1^2) u_x - ru \\ &+ \tfrac{1}{2}\sigma_2^2 u_{yy} + (r - \delta_2 - \tfrac{1}{2}\sigma_2^2) u_y + \rho\sigma_1\sigma_2 u_{xy} = 0\,.\end{aligned}$$

What are the matrix D and the vector b such that we arrive at (2)?

Exercise 12

In the three-dimensional (x,y,w)-space let the plane $w(x,y) = c_1 + c_2\,x + c_3\,y$ interpolate the three points $(x_i,\, y_i,\, w_i)$, $i = 1,2,3$. Show

$$\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}.$$

By inversion, establish a formula for $\nabla w = (c_2, c_3)^{tr}$.

Exercise 13

Consider the domain $\mathcal{D} := \{(x, y) \mid x \geq 0,\ y \geq 0,\ 1 \leq x + y \leq 2\}$ tiled by 12 triangles $\mathcal{D}_k$, where triangles and nodes are numbered as in the figure below.

a) Set up the index set $\mathcal{I}$ with entries $\mathcal{I}_k = \{i_k, j_k, l_k\}$, which assigns node numbers to the kth triangle for $1 \leq k \leq 12$.

b) Formulate the assembling algorithm that builds up the global matrix out of the local matrices

$$\begin{pmatrix} s_{11}^{(k)} & s_{12}^{(k)} & s_{13}^{(k)} \\ s_{21}^{(k)} & s_{22}^{(k)} & s_{23}^{(k)} \\ s_{31}^{(k)} & s_{32}^{(k)} & s_{33}^{(k)} \end{pmatrix}$$

for a general index set $\mathcal{I}$ and $1 \leq k \leq m$.

c) The example of the figure leads to a banded stiffness matrix. What is the bandwidth?

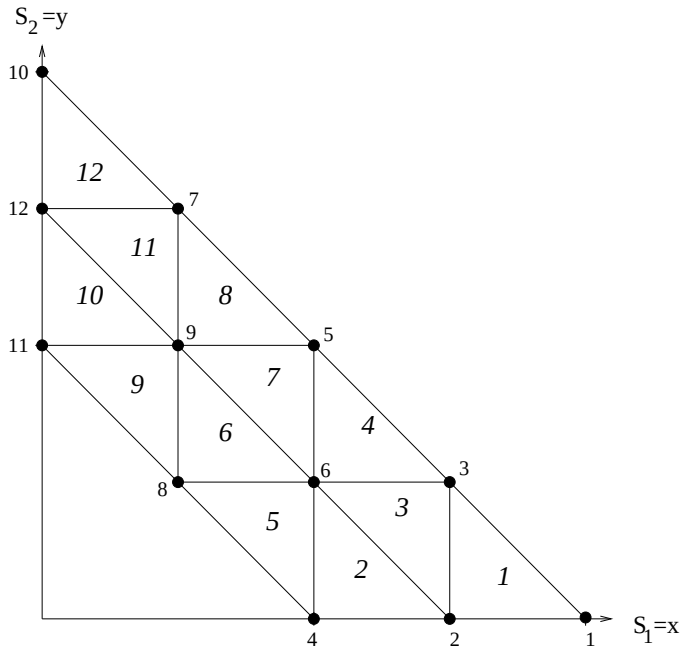

Figure: Specific triangulation and numbering

Besprechung dieses Blattes voraussichtlich am 4.2.2014.

Klausur: Mittwoch, 19.2.2014, 14-15 Uhr in H IV