Project A5

Advanced numerical methods for correlated quantum gases

Understanding dynamics of correlated quantum systems away from equilibrium is one of the main challenges of many-body theory. The goals of the project are to develop, to implement, and to apply numerical tools such as the time-evolving block decimation scheme (TEBD) for the simulation of dynamical problems of strongly interacting 1D systems, ranging from dynamical response in the ground state, to non-equilibrium dynamics and steady states of driven systems, In particular we investigate time-dependent phenomena in 1D bosonic lattice models and inhomogeneous continuous gases, as well as lattice systems with long-range interactions. Another open problem of strongly correlated systems studied within the SFB involves frustrated spin systems. To better understand the role of frustration we develop numerical techniques such as extensions of the density matrix renormalization group (DMRG) to two spatial dimensions. The techniques have been applied to the following problems:

Dynamical correlation functions in 1D gases

Dynamic correlation functions are increasingly important for ultra-cold atomic gases especially with the new possible time- and position-resolved setup in project A9 (Ott). In condensed matter physics such correlation functions have long been used for energy resolved expectation values such as the local density of states and the dynamical spin structure factor. Building on past experience in the numerical simulation of the dynamics of continuous 1D quantum gases using TEBD [1,2] we study the dynamical response of a one-dimensional Bose gas in the crossover regime between a weakly interacting (quasi-condensate) and a strongly interacting (Tonks-Girardeau) gas. This is done in close collaboration with the experimental project A9 (Ott).

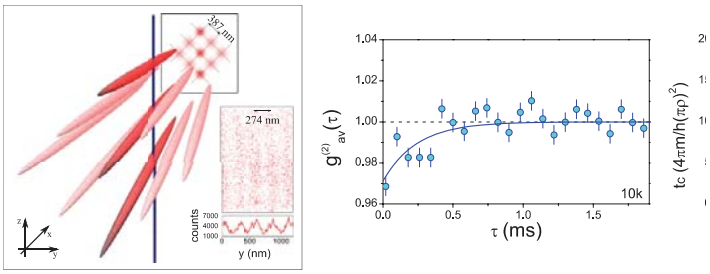

Figure 1: *left:* Experimental set-up for space and time resolved measure-ments of two-point correlations. *center:* Measured (dots) and calculated (line) two-time correlation $g^{(2)}(\tau)$ for strong interactions.

[1] Dominik Muth, Bernd Schmidt and Michael Fleischhauer, New J. Phys. **12**, 083065 (2010).

[2] Dominik Muth and Michael Fleischhauer, Phys. Rev. Lett. **105**, 150403 (2010).

Ordered states in optically driven Rydberg gases and Rydberg polaritons

Another focus of our research is the interplay of radiation with Rydberg atoms, which interact via long-range dipolar or van-der Waals forces. The dipolar interaction may favor an ordered state between the atoms, but it is far from clear how such a state can be prepared and if it remains stable. We study different systems where the radiation is used to (i) induce superradiance in Rydberg atoms confined to a cavity [3], (ii) to create an ordered state by storing a coherent pulse of photons in an atomic gas [4], or (iii) to continuously drive a transition to a Rydberg state in the presence of losses in one and two spatial dimensions [5].

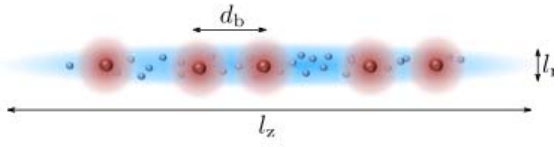

Figure 2: Van-der-Waals interactions between Rydberg atoms suppress excitations of atoms into Rydberg states within the so-called Blockade radius. This can give rise to different kinds of ordered states.

[3] X.-F. Zhang, Q. Sun, Y.-C. Wen, W.-M. Liu, S. Eggert, A.-C. Ji , Phys. Rev. Lett. **110**, 090402 (2013).

[4] J. Otterbach, M. Moos, D. Muth, M. Fleischhauer, Phys. Rev. Lett. **111**, 113001 (2013).

[5] M. Hoening, W. Abdussalam, M. Fleischhauer, and T. Pohl, Phys. Rev. A **90**, 021603 (R) (2014).

Quantum destillation in the Bose-Hubbard model

An important subject of our investigations is the dynamics of inhomogeneous many-body systems, in particular at the spatial boundary between different phases. Using TEBD we studied the dynamics of finite Mott-insulating cluster of boson pairs (doublons) in the 1D Bose Hubbard model to investigate the bosonic analogue of quantum distillation, first suggested for fermions [6]. For sufficiently strong repulsive interactions localized doublons of bosons are stable due to the absence of decay channels in a lattice. In contrast to fermions, the attraction between bosonic doublons results in stable clusters. On the other hand, defects in the form of singly occupied sites (holons) can freely propagate to the cluster boundary and can evaporate into the surrounding space leaving behind a doublon cluster of reduced size and with reduced entropy.

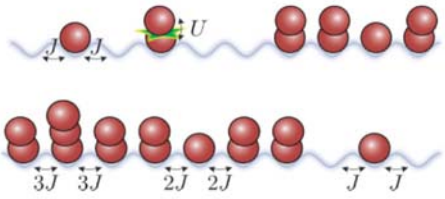

Figure 3: *top:* Finite cluster of n=2 Mott insulator in the Bose-Hubbard model. *bottom:* holons can freely propagate in the cluster and evaporate into free space at the border if they are in the right kinetic energy range. Additional triplons facilitate thermalization of holons

[6] F. Heidrich-Meisner et al., Phys. Rev. A **80**, 041603 (R) (2009).

Hard core bosons on the triangular lattices with negative hopping

Hard-core bosons on the triangular lattice were one of the first model systems where supersolidity was unambiguously shown using quantum Monte Carlo simulations [7,8]. Theoretically even more interesting is the case when the hopping can be tuned to negative values [9], which induces additional kinetic frustration. In this case the model is equivalent to the famous XXZ model on the triangular lattice. Unfortunately, quantum Monte Carlo simulations suffer from the sign problem, so we set out to provide a two-dimensional (2D) DMRG simulations for the phase diagram as a function of chemical potential and negative hopping strength (corresponding to the field and the xy-coupling in the XXZ model) [10].

[7] X.-F. Zhang, R. Dillenschneider, Y. Yu, and S. Eggert, Phys. Rev. B **84**, 174515 (2011).

[8] S. Wessel and M. Troyer, Phys. Rev. Lett. **95**, 127205 (2005).

[9] A. Eckardt et al. Europhys. Lett. **89**, 10010 (2010).

[10] Daniel Sellmann, Xue-Feng Zhang, and Sebastian Eggert, arXiv:1403.0008 preprint (2014).

Phase diagram of the extended Hubbard model on a triangular lattice

In collaboration with project B7 (Gros) we analyze the phase diagram of the triangular extended Hubbard model as a function of filling [11]. This model is relevant for describing the rich and complex behavior of organic conductors, such as charge transfer salts [12], where molecules are arranged on an anisotropic triangular lattice with incommensurate filling. Next to the relevance for this class of materials, the triangular Hubbard model is also interesting from a fundamental point of view as a frustrated system, where both spin and charge degrees exist in a complex interplay, while most previous studies have focused on spin-like frustration [7,8,13], where charge degrees of freedom only play a passive role. Accordingly, the phase diagram in the extended Hubbard is rather rich, containing phases of simultaneous charge order, spin order and conductivity [11].

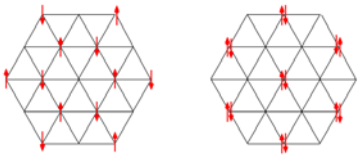

Figure 4: Different types of charge ordered states in the triangular extended Hubbard model.

[11] Luca F. Tocchio, Claudius Gros, Xue-Feng Zhang, and Sebastian Eggert, arXiv:1402.3160, Phys. Lett. in press (2014).

[12] C. Hotta, J. Phys. Soc. Jpn. 72, 840 (2003).

[13] C. Hotta and N. Furukawa, Phys. Rev. B 74, 193107 (2006).