Project B3

Correlations in antiferromagnets

The role of this project within the Transregio is to analyze correlations in antiferromagnets which are tunable by the chemical composition or the magnetic field in order to identify interesting new phases and quantum critical points. The study of phase transitions and strongly correlated behavior with the help of coupled spin systems has a long tradition ever since the early days of quantum mechanics. Today antiferromagnetic spin systems play a central role in many recent trends of theoretical condensed matter, especially in the context of studying quantum critical points and quantum phase transitions and in the search of new phases, including topological phases, supersolid phases, and spin-liquid phases. Many of our research activities on spin systems have a natural overlap to the ultra-cold gases community, since hard-core bosons on a lattice are theoretically equivalent to the XXZ model where the xy-coupling corresponds to negative hopping and the nearest neighbor interaction maps to the z-coupling. The following topics are examples of recent research activities:

1.) The frustrated $J_1 - J_2$ Heisenberg model

In the search of exotic quantum states and quantum phase transitions, frustrated antiferromagnetic systems have increasingly become the center of attention [1,2]. The competing interactions between spins potentially lead to a large entropy even at low temperatures, which together with quantum fluctuations may give rise to quantum phases with unconventional or topological order parameters [3–5]. In particular, the so-called spin-liquid state without long-range order of a conventional (local) order parameter has been much discussed in the literature ever since Anderson related this phase to high-temperature superconductivity using the famous J_1-J_2 Heisenberg model [6]. In this model antiferromagnetic couplings J_1 on a square lattice compete with diagonal couplings J_2, which leads to a well established intermediate phase for $0.4J_1 \lesssim J_2 \lesssim 0.6J_1$ [7]. However, to this date the nature of this intermediate phase is much debated to be dimerized [8–10] or a spin liquid [11, 12]. Since numerical work remains very difficult due to the extreme length scales that are required to demonstrate spin-liquid behavior [13], we have now used an analytical coupled chain renormalization group treatment of a more general anisotropic J_1-J_2-J_1'-J_2' model [14], which takes the effect of frustration into account within each chain before they are coupled. For very weak interchain couplings this method gives exact results and clearly predicts an intermediate dimerized phase. Using the renormalization group equations for larger values indicates that the dimer phase extends all the way to the isotropic $J_1 - J_2$ point as shown in Fig. 1, which was also confirmed by numerical simulations on the corresponding ladder model. This work is also an example of how the coupling of lower dimensional components leads to new physical behavior, which is also central for the following topic.

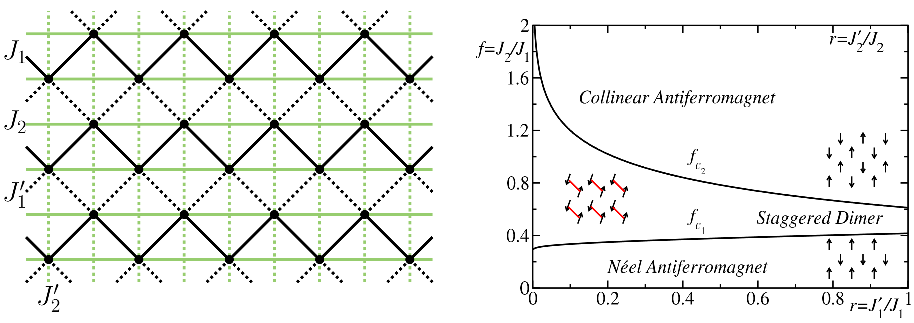

Figure 1: The generalized anisotropic J_1-J_2-J_1'-J_2' antiferromagnet together with the proposed phase diagram according to renormalization group treatment (from Ref. [14]).

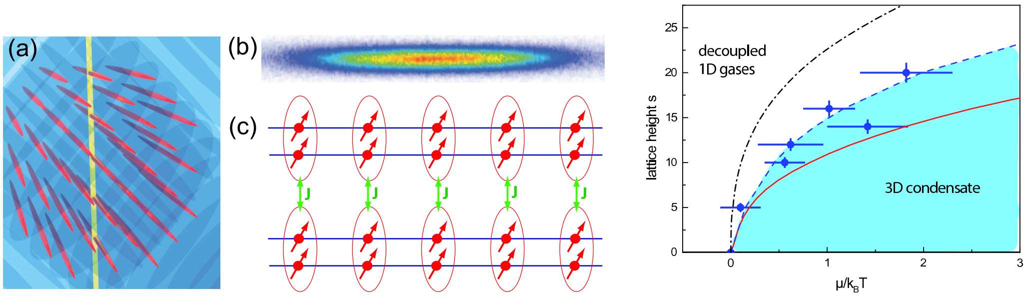

Figure 2: (a) 1D Bose gases (red) in a 2D optical lattice (blue) with finite tunneling coupling were imaged with an electron beam (yellow) (b) *In situ* image of the density distribution from which the line profiles are extracted. (c) Theoretically equivalent coupled ladders which show Bose-Einstein condensation of triplons above a critical magnetic field. Right: Phase-diagram as a function of chemical potential μ and lattice depth s. The dashed-dotted black line is the prediction of Ref. [19], the solid red line is the prediction of an anisotropic Bose-Einstein condensation theory (from Ref. [20]).

2.) Dimensional transition of coupled chains

The transition of weakly coupled one dimensional systems ("chains") to three dimensional behavior has recently been demonstrated and analyzed in experiments on coupled spin ladder systems in a magnetic field [15, 16]. Theoretically such a dimensional quantum phase transition of coupled ladders and chains can be described in a chain mean-field approach [17–19] or alternatively by a condensation of hard-core bosons. In a collaboration with experiments in project A9 (Ott) it was now possible to demonstrate that the analogous behavior can also be analyzed as a function of coupling strength using ultra-cold coupled bosonic chains [20]. The experimental setup and the resulting phase diagram are shown in Fig. 2. This analysis shows that the chain mean-field theory clearly overestimates the condensate region, while using an anisotropic version of the Bose-Einstein condensation theory is also quantitatively incorrect. We intend to use numerical simulations together with Luttinger liquid calculations which include higher order corrections to resolve this issue.

3.) Magnetocaloric effect near quantum criticality in spin dimer systems

Spin dimer systems are a promising playground for the detailed study of quantum phase transitions.

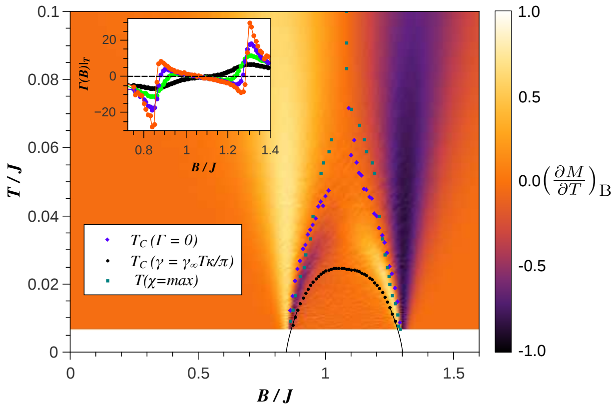

Figure 3: T-B phase diagram for the dimerized columnar square lattice. The surface plot shows the change of magnetisation with respect to the temperature. The green squares show the maximum of the susceptibility and the violet diamonds indicate the zero of the cooling rate. The black dots are the BKT transition temperature.

In many cases it is sufficient to use the magnetic field as the tuning parameter in order to reach interesting non-trivial critical points. Depending on the temperature it is in principle possible to observe a crossover from the characteristic scaling near the critical point to the behavior of a finite-temperature phase transition. In order to quantitatively demonstrate those effects and inspired by recent experiments in project B1 (Wolf/Lang) we compare numerical quantum Monte Carlo simulations with analytical calculations on strongly interacting boson systems in order to analyze several different physical quantities in spin dimer systems, namely the susceptibility, the magneto-caloric effect and the helicity modulus as shown in Fig. 3. This work is done in collaboration with project A8 (Kopietz). The phase transitions (quantum and finite temperature) are manifest in the characteristic scaling behavior near critical points. However, a logarithmic behavior, which had been predicted for 2D systems [23] could not be confirmed yet.

4.) Chiral edge states with fractional excitation in the Kagome lattice
Frustrated systems are not only interesting in order to establish new phases as mentioned above, but also exotic excitations are possible [24] such as Dirac strings in a spin-ice [25, 26], and fractional charges in Kagome lattice antiferromagnets [27]. For the XXZ model on a Kagome lattice with negative xy-coupling we now showed that the fractional spins localize near edges and are connected by quantum strings of resonant configurations [28]. This results in a new edge-liquid phase, which is characterized by a finite edge susceptibility but no long range spin stiffness. While fractional excitations at edges have long been known in 1D systems, such as the spin-1/2 degrees of freedom in a spin-1 chain [29], we believe that such an effect has so far been unknown in 2D.

References

[1] T.-H. Han, J. Helton, S. Chu, D. Nocera, J. Rodriguez-Rivera, C. Broholm, and Y. Lee, Nature **492**, 406 (2012).

[2] L. Balents, Nature **464**, 199 (2010).

[3] H.-C. Jiang, Z. Wang, and L. Balents, Nat. Phys. **8**, 902 (2012).

[4] A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006).

[5] M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006).

[6] P.W. Anderson, Science **235**, 1196 (1987).

[7] T. Einarsson and H.J. Schulz, Phys. Rev. B **51**, 6151 (1995); H.J. Schulz and T.A.L. Ziman, Europhys. Lett. **18**, 355 (1992).

[8] S-S. Gong, W. Zhu, D.N. Sheng, O.I. Motrunich, M.P.A. Fisher, Phys. Rev. Lett. **113**, 027201 (2014).

[9] M.P. Gelfand, Phys. Rev. B **42**, 8206 (1990).

[10] J. Sirker, Z. Weihong, O. P. Sushkov, and J. Oitmaa, Phys. Rev. B **73**, 184420 (2006).

[11] H.-C. Jiang, H. Yao, and L. Balents, Phys. Rev. B **86**, 024424 (2012).

[12] W-J. Hu, F. Becca, A. Parola, and S. Sorella, Phys. Rev. B **88**, 060402 (2013); T. Li, F. Becca, W. Hu, and S. Sorella, Phys. Rev. B **86**, 075111 (2012).

[13] A. W. Sandvik, Phys. Rev. B **85**, 134407 (2012).

[14] Alexandros Metavitsiadis, Daniel Sellmann, and Sebastian Eggert, Phys. Rev. B **89**, 241104(R) (2014)

[15] Ch. Rüegg *et al.*, Phys. Rev. Lett. **101**, 247202 (2008).

[16] B. Thielemann *et al.*, Phys. Rev. B **79**, 020408(R) (2009).

[17] T. Giamarchi and A.M. Tsvelik, Phys. Rev. B **59**, 11398 (1999).

[18] I. Affleck, M.P. Gelfand, and R.R.P. Singh, J. Phys. A **27**, 7313 (1994); S. Eggert, I. Affleck, and M.D.P. Horton, Phys. Rev. Lett. **89**, 047202 (2002).

[19] T. Giamarchi, C. Rüegg, and O. Tchernyshyov, Nature Phys. **4**, 198 (2008); M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and M. Rigol, Rev. Mod. Phys. **83**, 1405 (2011).

[20] A. Vogler, R. Labouvie, S. Barontini, S. Eggert, V. Guarrera, and H. Ott, Phys. Rev. Lett. in press (2014).

[21] N. Sedlmayr, D. Morath, I. Affleck, J. Sirker, and S. Eggert, Phys. Rev. B **89**, 045133 (2014).

[22] N. Sedlmayr, J. Ohst, I. Affleck, J. Sirker, and S. Eggert, Phys. Rev. B **86**, 121302(R) (2012)

[23] S. Sachdev, T. Senthil, and R. Shankar, Phys. Rev. B **50**, 258 (1994).

[24] T.-H. Han, J.S. Helton, S.Y. Chu, D. G. Nocera, J.A. Rodriguez-Rivera, C. Broholm, and Y.S. Lee, Nature **492**, 406 (2012)

[25] A.P. Ramirez, A. Hayashi, R.J. Cava, R.B. Siddharthan, and S. Shastry, Nature **399**, 333 (1999).

[26] C. Castelnovo, R. Moessner, and S. L. Sondhi, Nature **451**, 42 (2007)

[27] A. O'Brien, F. Pollmann, and P. Fulde, Phys. Rev. B **81**, 235115 (2010)

[28] Xue-Feng Zhang and Sebastian Eggert, Phys. Rev. Lett. **111**, 147201 (2013)

[29] I. Affleck, T. Kennedy, E.H. Lieb, and H. Tasaki, Phys. Rev. Lett. **59**, 799 (1987); T. Kennedy, J. Phys.: Condens. Matter **2**, 5737 (1990).