A family of local times studied as a Markov process

The classical Ray-Knight Theorem for a Brownian motion B describes the local times of B taken at the first time T_a that B hits a level a : they are distributed as a squared Bessel process of dimension 2. In this talk I will consider a path-valued Markov process Z_a defined by $Z_a(y) = l(T_a, a-y)$ for $y \geq 0$, where $l(.,.)$ are the local times of B.

At first sight the evolution of Z seems quite simple: it makes jumps that correspond to the excursions that B makes below its running maximum. But studying Z with the tools of Markov process theory (duality, excessive functions, Revuz measures) produces some interesting insights. In particular I will explain a conjecture that if $0 \leq d < 10$, then there exists a random choice of the level a so that $(l(T_a, a-y),\ 0 \leq y \leq a)$ is distributed not as a squared Bessel process of dimension 2, but as a squared Bessel process of dimension d. On the other hand if $d \geq 10$, then no such random level exists.