A fundamental open question in the theory of linear programming is whether there exists a pivot rule such that the Simplex Method with that rule runs in polynomial time. Part of the difficulty in studying this question is that the space of all pivot rules is not easy to define. Instead, we introduce a tractable yet robust subclass called *normalized weight pivot rules.* The normalization serves a role similar to the norm in the steepest edge pivot rule, and the weight is a vector that works as an auxiliary objective function as with the shadow vertex pivot rule. For a fixed LP, a normalized weight pivot rule determines a unique choice of outgoing edge for each vertex on the graph of the associated polytope and thus an arborescence. Our main results are finding two polytopes that describe how these arborescences vary as we vary either the weight or objective function. We relate these polytopes to other known constructions and show these polytopes yield new realizations of polytopes in algebraic combinatorics including the Stanley-Pitman polytope, associahedron, and permutahedra of types A and B.