Vector bundles on curves over $\mathbb{C}_p$

Annette Werner

1. Introduction. This paper is a report on joint work with Christopher Deninger published in [De-We1], [De-We2] and [De-We3]. We define a certain class of vector bundles on p-adic curves which can be endowed with parallel transport along étale paths. In particular, all those bundles induce representations of the algebraic fundamental group.

This article is intended as a survey, explaining some results more leisurely than in the original papers. We focus mainly on the definitions and constructions in [De-We2], outlining the ideas rather than reproducing the formal proofs.

The last section deals with Mumford curves. We explain results of Herz in [He] which relate our construction to the paper [Fa1].

Note that in [Fa2], Faltings develops a p-adic non-abelian Hodge theory, which in some aspects is more general than the present theory.

2. Complex vector bundles. Let X be a compact Riemann surface with base point $x \in X$. Then every complex representation $\rho : \pi_1(X, x) \to \mathrm{GL}_r(\mathbb{C})$ of the fundamental group gives rise to a flat vector bundle E_ρ on X, i.e. a vector bundle with locally constant transition function. Namely, let $\pi : \tilde{X} \to X$ be the universal covering of X. Then E_ρ is defined as the quotient of the trivial bundle $\tilde{X} \times \mathbb{C}^r$ by the $\pi_1(X, x)$-action given by combining the natural action of $\pi_1(X, x)$ on the first factor with the action induced by ρ on the second factor. Conversely, if E is a flat vector bundle on X, its pullback π^*E is trivial as a flat bundle on $\tilde{X}$, i.e. $\pi^*E \simeq \mathbb{C}^r \times \tilde{X}$. Since $\pi_1(X, x)$ acts in a natural way on the pullback bundle π^*E, it also acts on the right hand side, which gives a representation ρ of $\pi_1(X, x)$ on $\mathbb{C}^r$. This representation satisfies $E \simeq E_\rho$.

A similar construction gives for every flat bundle E and every continuous path γ from x to x' in X an isomorphism of parallel transport along γ from the fibre E_x of E in x to the fibre $E_{x'}$. Namely, choose a point y in the universal covering $\tilde{X}$ over x. Then γ can be lifted to a continuous path in $\tilde{X}$ starting in y. The endpoint of the lifted path is a point y' in $\tilde{X}$ lying over x'. Since π^*E is trivial, there is a trivial parallel transport $(\pi^*E)_y \xrightarrow{\sim} (\pi^*E)_{y'}$. Since $\pi(y) = x$ and $\pi(y') = x'$, there are natural isomorphisms $(\pi^*E)_y \xrightarrow{\sim} E_x$ and $(\pi^*E)_{y'} \xrightarrow{\sim} E_{x'}$. Putting all these isomorphisms together, parallel transport along γ is given by

$$E_x \xrightarrow{\sim} (\pi^*E)_y \xrightarrow{\sim} (\pi^*E)_{y'} \xrightarrow{\sim} E_{x'}.$$

Regarding E_ρ as a holomorphic bundle on X, a theorem of Weil [Weil] says that a holomorphic bundle E on X is isomorphic to some E_ρ (i.e. E comes from a representation of $\pi_1(X, x)$) if and only if $E = \bigoplus_i E_i$ with indecomposable subbundles E_i of degree zero. A famous result by Narasimhan and Seshadri [Na-Se] says that a holomorphic vector bundle E of degree zero on X is stable if and only if E is isomorphic to E_ρ for some irreducible unitary representation ρ. Hence a holomorphic vector bundle comes from a unitary representation ρ if and only if it is of the form $E = \bigoplus_i E_i$ for stable (and hence indecomposable) subbundles of degree zero.

3. Fundamental groups of p-adic curves. It is a natural question to look for a p-adic analogue of the results described in the previous section. We denote by $\mathbb{C}_p$ the completion of an algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}_p$. Besides, $\overline{\mathbb{Z}}_p$ and $\mathfrak{o}$ denote the rings of integers in $\overline{\mathbb{Q}}_p$ and $\mathbb{C}_p$, respectively. By $k = \overline{\mathbb{F}}_p$ we denote the residue field of $\overline{\mathbb{Z}}_p$ and $\mathfrak{o}$.

We call any purely one-dimensional separated scheme of finite type over a field a curve. Let X be a smooth, projective and connected curve over $\overline{\mathbb{Q}}_p$ and $X_{\mathbb{C}_p}$ its base change to $\mathbb{C}_p$. We are looking for a relation between vector bundles on $X_{\mathbb{C}_p}$ and representations of the fundamental group of X.

First we have to clarify what we mean by fundamental group. Of course, in the algebraic setting there is no topological fundamental groups defined with closed paths. However, there is an algebraic fundamental group $\pi_1(X, x)$ for a base point $x \in X(\mathbb{C}_p)$. It is defined as the group of automorphisms of the fibre functor F_x. This fibre functor F_x maps a finite étale covering Y of X to the set of $\mathbb{C}_p$-valued points of Y lying over x. Hence an automorphism of F_x induces in a functorial way for every finite étale covering $Y \to X$ a permutation of the points in the fibre over x. Note that on a Riemann surface X, a closed topological path γ on X induces such an automorphism $\gamma : F_x \xrightarrow{\sim} F_x$ of the fibre functor. Namely, for every finite étale cover $Y \to X$ and every point $y \in Y$ over x we can lift γ to a continuous path in Y starting in y. Its endpoint y' also lies over x, and one can define γ by mapping $y \in F_x(Y)$ to $y' \in F_x(Y)$.

Since the algebraic fundamental group involves only the finite étale coverings, it is in fact analogous to the profinite completion of the topological fundamental group on Riemann surfaces.

There is also an analogue of non-closed paths. For two points x and x' in $X(\mathbb{C}_p)$ we call any isomorphism $F_x \xrightarrow{\sim} F_{x'}$ of the corresponding fibre functors an étale path from x to x. Such an étale path associates for all finite, étale $Y \to X$ to every point in the fibre of Y over x a point in the fibre of Y over x'.

The étale fundamental groupoid of $\Pi_1(X)$ of X is defined as the category such that the points in $X(C_p)$ are the objects and such the set of morphisms from $x \in X(\mathbb{C}_p)$ to $x' \in X(\mathbb{C}_p)$ is the set of étale paths from x to x', i.e. the set of isomorphisms of fibre functors $F_x \xrightarrow{\sim} F_{x'}$.

4. Finite vector bundles. The algebraic fundamental group only involves finite étale coverings, hence in the algebraic setting there is no universal covering. Therefore we can imitate the constructions on Riemann surfaces in the p-adic situation only for finite vector bundles, i.e. for vector bundles E on $X_{\mathbb{C}_p}$ such that there is a finite étale covering $\pi : Y_{\mathbb{C}_p} \to X_{\mathbb{C}_p}$ for which π^*E is trivial .

Namely, for every finite vector bundle E on $X_{\mathbb{C}_p}$ we choose a finite, étale and Galois covering $\pi : Y_{\mathbb{C}_p} \to X_{\mathbb{C}_p}$ trivializing E and a point y in $Y_{\mathbb{C}_p}(\mathbb{C}_p)$ lying over x. Then there is a short exact sequence

$$0 \to \pi_1(Y_{\mathbb{C}_p}, y) \to \pi_1(X_{\mathbb{C}_p}, x) \to \mathrm{Gal}(Y_{\mathbb{C}_p}/X_{\mathbb{C}_p}) \to 0.$$

Besides, since π^*E is trivial, the fibre map $\Gamma(Y_{\mathbb{C}_p}, \pi^*E) \to (\pi^*E)_y$ is an isomorphism. As a pullback bundle, π^*E and also its set of global sections $\Gamma(Y_{\mathbb{C}_p}, \pi^*E)$ carries a natural $\mathrm{Gal}(Y_{\mathbb{C}_p}/X_{\mathbb{C}_p})$-action. Via the fibre isomorphism, this action induces a $\mathrm{Gal}(Y_{\mathbb{C}_p}/X_{\mathbb{C}_p})$-action on $(\pi^*E)_y$, which can be identified with the fibre E_x of E in x. Therefore we have defined a representation

$$\pi_1(X, x) = \pi_1(X_{\mathbb{C}_p}, x) \to \mathrm{Gal}(Y_{\mathbb{C}_p}/X_{\mathbb{C}_p}) \to \mathrm{Aut}(E_x).$$

In fact, this construction works for curves over arbitrary fields. In [La-Stu], Lange and Stuhler investigate finite vector bundles on a smooth, projective curve C over a field of characteristic p. Let us denote by F the absolute Frobenius on C, defined by the p-power-map on the structure sheaf. By [La-Stu], 1.4 a vector bundle E on C is finite if and only if for a suitable power F^n we have $F^{n*}E \xrightarrow{\sim} E$.

5. A bigger category of vector bundles. Let us again consider a smooth, projective and connected curve X over $\overline{\mathbb{Q}}_p$. We are interested in vector bundles on $X_{\mathbb{C}_p}$. Here we denote by $X_{\mathbb{C}_p}$ the base change of X with $\mathbb{C}_p$. This kind of notation for base changes will be used throughout this paper.

We have seen that finite vector bundles on $X_{\mathbb{C}_p}$ give rise to representations of the fundamental group. However, in this way we only get representations factoring over a finite quotient of $\pi_1(X, x)$.

The main idea for the construction of representations for a more general class of vector bundles is the following: We consider vector bundles with integral models (in a sense to be made precise below) which are " finite modulo p^n" for all n. A similar construction as the one for finite bundles then gives for all n representations of $\pi_1(X, x)$ modulo p^n, i.e. over $\mathfrak{o}/p^n\mathfrak{o}$. In the limit we get a representation of $\pi_1(X, x)$ over $\mathfrak{o}$ which we can tensor with $\mathbb{C}_p$. In fact, we will more generally define parallel transport along étale paths.

To be more precise, let us call any finitely presented, flat and proper $\overline{\mathbb{Z}}_p$-scheme (repectively $\mathfrak{o}$-scheme) with generic fibre X (respectively $X_{\mathbb{C}_p}$) a model of X (respectively $X_{\mathbb{C}_p}$).

Note that any model of X descends to a finite extension of $\mathbb{Z}_p$ and is irreducible and reduced by [Liu], 4.3.8.

We assume that the curves and their models are finitely presented over $\overline{\mathbb{Q}}_p$, repectively $\overline{\mathbb{Z}}_p$, so that they can be descended to a finite extension of $\mathbb{Q}_p$, respectively $\mathbb{Z}_p$. We need this descent to a noetherian situation in some arguments. Our vector bundles and their models however live over $\mathbb{C}_p$, respectively $\mathfrak{o}$.

Definition 1 *We denote by $\mathfrak{B}_{X_{\mathbb{C}_p}}$ the full subcategory of all vector bundles on $X_{\mathbb{C}_p}$ for which there is a model $\mathcal{X}$ of X over $\overline{\mathbb{Z}}_p$ and a vector bundle $\mathcal{E}$ on $\mathcal{X}_{\mathfrak{o}}$ with generic fibre E such that for all natural numbers $n \geq 1$ there is a finitely presented proper $\overline{\mathbb{Z}}_p$-morphism $\pi : \mathcal{Y} \to \mathcal{X}$ with the following two properties:*

i) The generic fibre $\pi_{\overline{\mathbb{Q}}_p} : Y = \mathcal{Y} \otimes \overline{\mathbb{Q}}_p \to X$ is finite and étale

ii) The pullback bundle $\pi_{\mathfrak{o}}^* \mathcal{E}$ becomes trivial on $\mathcal{Y}_{\mathfrak{o}}$ after base change with $\mathfrak{o}/p^n\mathfrak{o}$.

Hence $\mathfrak{B}_{X_{\mathbb{C}_p}}$ can be viewed as the category of all vector bundles on $X_{\mathbb{C}_p}$ which are in some sense finite modulo all p^n. Note however that the special fibre of the coverings $\pi : \mathcal{Y} \to \mathcal{X}$ will in general be neither finite nor étale. Only the generic fibre has these properties.

Note that every vector bundle E on $X_{\mathbb{C}_p}$ can be extended to a bundle on a suitable model of $X_{\mathbb{C}_p}$, see e.g. [De-We2], theorem 5. Hence the important point in definition 1 is the existence of the coverings π. Although we omit it in our notation, π depends of course on n.

This definition of the category $\mathfrak{B}_{X_{\mathbb{C}_p}}$ is useful for the construction of parallel transport, as we will see in the next section. However, it is difficult to check whether a bundle fulfills the conditions in definition 1. In sections 10 and 11 we give a more intrinsic characterizations of $\mathfrak{B}_{X_{\mathbb{C}_p}}$ (and also of a bigger category of bundles to be defined below).

6. Parallel transport on bundles in $\mathfrak{B}_{X_{\mathbb{C}_p}}$. Let us fix a bundle E in $\mathfrak{B}_{X_{\mathbb{C}_p}}$. For all n, the morphism $\pi : \mathcal{Y} \to \mathcal{X}$ descends to a morphism $\pi_R : \mathcal{Y}_R \to \mathcal{X}_R$ over a discrete valuation ring R finite over $\mathbb{Z}_p$. If R is chosen big enough, there is a semistable R-curve $\mathcal{Y}'$ over $\mathcal{Y}$ with geometrically connected generic fibre such that the induced map $\pi' : \mathcal{Y}' \otimes \overline{\mathbb{Z}}_p \to \mathcal{X}$ also has the properties i) and ii) in definition 1, see e.g. [De-We2], theorem 1. Hence we can assume that all the $\mathcal{Y}$ in definition 1 are semistable with connected generic fibres. It follows that the structure maps $\lambda : \mathcal{Y} \to \operatorname{Spec} \overline{\mathbb{Z}}_p$ are cohomologically flat in dimension zero. This means that the formation of $\lambda_* \mathcal{O}_{\mathcal{Y}}$ commutes with arbitrary base changes. As a consequence, we have $(\lambda \otimes R)_* \mathcal{O}_{\mathcal{Y} \otimes R} = \mathcal{O}_R$ for every $\overline{\mathbb{Z}}_p$-algebra R.

Now let us fix two points x and x' in $X(\mathbb{C}_p)$ and an étale path γ from x to x', i.e. an isomorphism of fibre functors $F_x \xrightarrow{\sim} F_{x'}$. Besides, we fix some $n \geq 1$. Then there is a morphism $\pi : \mathcal{Y} \to \mathcal{X}$ as in definition 1 with a semistable $\mathcal{Y}$. We fix some y in $\mathcal{Y}(\mathbb{C}_p)$ with $\pi(y) = x$. The fibre functor γ maps y to a point $y' \in \mathcal{Y}(\mathbb{C}_p)$ with $\pi(y') = x'$. The points y and y' can be extended to $\mathfrak{o}$-rational points on the proper model $\mathcal{Y}$, and x and x' can be extended to $\mathfrak{o}$-rational point $x_{\mathfrak{o}}$ and $x'_{\mathfrak{o}}$ on $\mathcal{X}$.

By x_n, x'_n, y_n respectively y'_n we denote the induced $\mathfrak{o}_n$-rational points, where we put $\mathfrak{o}_n = \mathfrak{o}/p^n\mathfrak{o} = \overline{\mathbb{Z}}_p/p^n\overline{\mathbb{Z}}_p$. Besides, we write $\pi_n : \mathcal{Y}_n \to \mathcal{X}_n$ for the base change of π with $\mathfrak{o}_n$. In particular, $\mathcal{X}_n = \mathcal{X} \otimes_{\overline{\mathbb{Z}}_p} \mathfrak{o}_n$ and $\mathcal{Y}_n = \mathcal{Y} \otimes_{\overline{\mathbb{Z}}_p} \mathfrak{o}_n$. Let $\mathcal{E}_n = \mathcal{E} \otimes_{\mathfrak{o}} \mathfrak{o}_n$ be the induced vector bundle on $\mathcal{X}_n$.

For the $\overline{\mathbb{Z}}_p$-algebra $R = \mathfrak{o}_n$ the equality above gives $(\lambda \otimes \mathfrak{o}_n)_* \mathcal{O}_{\mathcal{Y}_n} = \mathfrak{o}_n$. This implies that the $\mathfrak{o}_n$-rational point $y_n : \operatorname{Spec}(\mathfrak{o}_n) \to \mathcal{Y}_n$ induces by pullback an isomorphism

$$y_n^* : \Gamma(\mathcal{Y}_n, \mathcal{O}_{\mathcal{Y}_n}) \xrightarrow{\sim} \Gamma(\mathrm{Spec}\,(\mathfrak{o}_n), y_n^* \mathcal{O}_{\mathcal{Y}_n}) = \mathfrak{o}_n.$$

Since $\pi_n^* \mathcal{E}_n$ is trivial on $\mathcal{Y}_n$, pullback by y_n also induces an isomorphism

$$y_n^* : \Gamma(\mathcal{Y}_n, \pi_n^* \mathcal{E}_n) \xrightarrow{\sim} \Gamma(\mathrm{Spec}\,(\mathfrak{o}_n), y_n^* \pi_n^* \mathcal{E}_n) = \Gamma(\mathrm{Spec}\,(\mathfrak{o}_n), x_n^* \mathcal{E}_n) =: \mathcal{E}_{x_n}.$$

Similarly, pullback by y_n' induces an isomorphism

$$y_n'^* : \Gamma(\mathcal{Y}_n, \pi_n^* \mathcal{E}_n) \xrightarrow{\sim} \Gamma(\mathrm{Spec}\,(\mathfrak{o}_n), y_n'^* \pi_n^* \mathcal{E}_n) = \Gamma(\mathrm{Spec}\,(\mathfrak{o}_n), x_n'^* \mathcal{E}_n) =: \mathcal{E}_{x_n'}.$$

Now we define $\rho_{\mathcal{E},n}(\gamma)$ to be the composition

$$\rho_{\mathcal{E},n}(\gamma) = y_n'^* \circ (y_n^*)^{-1} : \mathcal{E}_{x_n} \xleftarrow{\sim} \Gamma(\mathcal{Y}_n, \pi_n^* \mathcal{E}_n) \xrightarrow{\sim} \mathcal{E}_{x_n'}.$$

Then the maps $\rho_{\mathcal{E},n}(\gamma)$ form a projective system. We denote by

$$\mathcal{E}_{x_\mathfrak{o}} = \varprojlim_n \mathcal{E}_{x_n}$$

the fibre of $\mathcal{E}$ over $x_\mathfrak{o}$ and by $\mathcal{E}_{x_\mathfrak{o}'}$ the fibre of $\mathcal{E}$ over $x_\mathfrak{o}'$. In the limit we get a map

$$\rho_{\mathcal{E}}(\gamma) : \mathcal{E}_{x_\mathfrak{o}} \to \mathcal{E}_{x_\mathfrak{o}'}.$$

Tensoring with $\mathbb{C}_p$ we finally get a "parallel transport" map

$$\rho_E(\gamma) : E_x \to E_{x'},$$

where E_x is the fibre of E in x. By construction, this isomorphism is continuous in the p-adic topology. In [De-We2], section 3, we show that the construction is independent of all choices.

7. Working outside a divisor on $X_{\mathbb{C}_p}$. The definition of parallel transport in the last section can be extended to open subcurves of X in the following way:

Let D be a divisor on X, and put $U = X \setminus D$. In the following, only the support of D plays a role. By $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ we denote the category of all vector bundles E on $X_{\mathbb{C}_p}$ such that there exists a model $\mathcal{X}$ of X and a vector bundle $\mathcal{E}$ on $\mathcal{X}_\mathfrak{o}$ with generic fibre E such that for all natural numbers $n \geq 1$ there is a finitely presented proper $\overline{\mathbb{Z}}_p$-morphism $\pi : \mathcal{Y} \to \mathcal{X}$ with the following two properties:

i) The generic fibre $\pi_{\overline{\mathbb{Q}}_p} : Y = \mathcal{Y} \otimes \overline{\mathbb{Q}}_p \to X$ is finite and its restriction to $\pi^{-1}_{\overline{\mathbb{Q}}_p}(U)$ is étale.

ii) The pullback bundle $\pi_{\mathfrak{o}}^* \mathcal{E}$ becomes trivial on $\mathcal{Y}_{\mathfrak{o}}$ after base change with $\mathfrak{o}/p^n\mathfrak{o}$.

Thus $\mathfrak{B}_{X_{\mathbb{C}_p}} = \mathfrak{B}_{X_{\mathbb{C}_p},\emptyset}$. Let γ be an étale path from $x \in U(\mathbb{C}_p)$ to $x' \in U(\mathbb{C}_p)$ in U, i.e. an isomorphim of fibre functors defined on the open curve U. The same construction as above gives for every bundle E in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ an isomorphism "of parallel transport"

$$\rho_E(\gamma) : E_x \to E_{x'}.$$

8. Properties of parallel transport. By [De-We2], theorem 22, the association $\rho : \gamma \mapsto \rho_E(\gamma)$ is functorial in γ, i.e. for $E \in \mathfrak{B}_{X_{\mathbb{C}_p},D}$ and étale paths γ from x to x' and γ' from x' to x'' we have $\rho_E(\gamma' \circ \gamma) = \rho_E(\gamma') \circ \rho_E(\gamma)$ as isomorphisms from E_x to $E_{x''}$.

Recall that for $U = X \backslash D$ the fundamental groupoid $\Pi_1(U)$ is the category with object set $U(\mathbb{C}_p)$ and étale paths as morphisms. In fact, for x and x' in $U(\mathbb{C}_p)$ the morphism set $\mathrm{Mor}(x, x') = \{$étale paths from x to $x' \} = \mathrm{Iso}(F_x, F_{x'})$ carries a natural topology, since it is profinite. A functor from $\Pi_1(U)$ to the category of finite-dimensional $\mathbb{C}_p$-vector spaces which is continuous on the morphism spaces is called a representation of $\Pi_1(U)$ on finite-dimensional $\mathbb{C}_p$-vector spaces.

With this terminology, for every vector bundle $E \in \mathfrak{B}_{X_{\mathbb{C}_p},D}$ the functor

$$\rho_E : \Pi_1(U) \to \{\text{finite} - \text{dimensional } \mathbb{C}_p\text{−vector spaces}\}$$

given by $x \mapsto E_x$ on objects and $\gamma \mapsto \rho_E(\gamma)$ on morphisms is a representation of $\Pi_1(U)$.

By [De-We2], theorem 28, we have

Theorem 2 *i) The association $E \mapsto \rho_E$ is functorial for morphisms in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$, exact and commutes with tensor products, duals, internal homs and exterior powers.*

ii) If $f : X \to X'$ is a morphism between smooth, projective, connected curves over $\overline{\mathbb{Q}}_p$, and D' is a divisor on X', then pullback of vector bundles induces a functor $f^* : \mathfrak{B}_{X'_{\mathbb{C}_p},D'} \to \mathfrak{B}_{X_{\mathbb{C}_p},f^*D'}$ which commutes with tensor *products, duals, internal homs and exterior powers. Besides, for every bundle* E *in* $\mathfrak{B}_{X'_{\mathbb{C}_p},D'}$ *we have* $\rho_{f^*E} = \rho_E \circ f_*$ *where* f_* *is the induced functor* $\Pi_1(X\backslash f^*D') \to \Pi_1(X'\backslash D')$ *on fundamental groupoids. Here we identify* $(f^*E)_x$ *with* $E_{f(x)}$.

iii) If $X = X_K \otimes_K \overline{\mathbb{Q}}_p$ *for some field* K *between* $\mathbb{Q}_p$ *and* $\overline{\mathbb{Q}}_p$*, then every element* σ *in* $\mathrm{Gal}(\overline{\mathbb{Q}}_p/K)$ *acts in a natural way on* $\mathfrak{B}_{X_{\mathbb{C}_p},D}$. *Besides,* σ *acts on* $\Pi_1(X\backslash D)$ *and on the category of finite-dimensional* $\mathbb{C}_p$*-vector spaces and hence on the category of representations of* $\Pi_1(X\backslash D)$. *The functor* ρ *commutes with these actions.*

9. Semistable bundles. Recall that for a vector bundle E on a smooth, projective and connected curve over a field k the slope is defined by $\mu(E) = \deg(E)/\mathrm{rk}(E)$. The bundle E is called semistable (respectively stable), if for all proper non-zero subbundles F of E the inequality $\mu(F) \leq \mu(E)$ (respectively $\mu(F) < \mu(E)$) holds.

Let E be a bundle in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ for some divisor D. By definition, there exists a model $\mathcal{X}$ of X, a vector bundle $\mathcal{E}$ on $\mathcal{X}_{\mathfrak{o}}$ extending E and a finitely presented, flat morphism $\pi : \mathcal{Y} \to \mathcal{X}$ which is generically finite and étale over $X\backslash D$ such that the special fibre of $\pi_{\mathfrak{o}}^*\mathcal{E}$ is trivial. Here we only use the condition for $n = 1$ in the definition of $\mathfrak{B}_{X_{\mathbb{C}_p},D}$.

Using descent to a suitable discrete valuation ring and an argument due to Raynaud (see [De-We2], theorem 13) one can show that triviality of the special fibre of $\pi_{\mathfrak{o}}^*\mathcal{E}$ implies that the generic fibre $\pi_{\mathbb{C}_p}^* E$ is semistable of degree zero on $\mathcal{Y} \otimes \mathbb{C}_p$. Since $\pi_{\mathbb{C}_p}$ is finite, E is also semistable of degree zero on $X_{\mathbb{C}_p}$.

10. A simpler description of $\mathfrak{B}_{X_{\mathbb{C}_p},D}$. It turns out that the existence of some $\pi : \mathcal{Y} \to \mathcal{X}$ as above such that $\pi_{\mathfrak{o}}^*\mathcal{E}$ has a trivial special fibre is also sufficient for a bundle to lie in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$. To be precise, by [De-We2], theorem 16 we have

Theorem 3 *A vector bundle* E *lies in* $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ *if and only if there is a model* $\mathcal{X}$ *of* X*, a vector bundle* $\mathcal{E}$ *on* $\mathcal{X}_{\mathfrak{o}}$ *extending* E *and a finitely presented, proper morphism* $\pi : \mathcal{Y} \to \mathcal{X}$ *which is generically finite and étale over* $X\backslash D$ *such that the special fibre of* $\pi_{\mathfrak{o}}^*\mathcal{E}$ *is trivial.*

Let E be a bundle in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ with a model $\mathcal{E}$ on $\mathcal{X}_{\mathfrak{o}}$, and let $\pi : \mathcal{Y} \to \mathcal{X}$ be as in theorem 3. The image $\pi(\mathcal{Y})$ is closed in $\mathcal{X}$ and contains the generic fibre, since π is generically finite. Since the model $\mathcal{X}$ is irreducible, π must be surjective. Let $\pi_k : \mathcal{Y}_k \to \mathcal{X}_k$ be its special fibre (recall that $k = \overline{\mathbb{F}}_p$ is the residue field of $\overline{\mathbb{Z}}_p$ and $\mathfrak{o}$). Let $C_1, \ldots, C_r$ be the irreducible components of $\mathcal{X}_k$ with their reduced structure. Since π is surjective, every C_ν is finitely dominated by an irreducible component D_ν of $\mathcal{Y}_k$. Hence the restriction of $\mathcal{E}_k$ to C_ν is trivialized by a finite covering (which of course in general is not étale). Let $\tilde{C}_\nu$ be the normalization of C_ν. Then also the pullback of $\mathcal{E}_k$ to the smooth projective k-curve $\tilde{C}_\nu$ is trivialized by a finite covering, namely the normalization of D_ν.

11. Strongly semistable reduction. Let us denote by F the absolute Frobenius in characteristic p.

Definition 4 *Let E be a vector bundle on a smooth, projective, connected curve C over a field of characteristic p. Then E is called strongly semistable if and only if for all $n \geq 0$ the pullback $F^{n*}E$ is semistable on C.*

Let E be a vector bundle in the category $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ with an extension $\mathcal{E}$ to a model $\mathcal{X}_\mathfrak{o}$ such that the special fibre of $\mathcal{E}$ becomes trivial after pullback to some $\mathcal{Y} \to \mathcal{X}$ as above. We have seen in the preceeding section that for all irreducible components C_ν of the special fibre $\mathcal{X}_k$ the pullback of $\mathcal{E}_k$ to the normalization $\tilde{C}_\nu$ of C_ν becomes trivial on a finite covering. As the trivial vector bundle is strongly semistable of degree zero, we deduce that the pullback of $\mathcal{E}_k$ to any $\tilde{C}_\nu$ is also strongly semistable of degree zero.

One of the main results in [De-We2] shows that this property is also sufficient for a bundle to lie in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$.

To be precise, we say that a vector bundle E on $X_{\mathbb{C}_p}$ has strongly semistable reduction of degree zero, if there is a model $\mathcal{X}$ of X and a vector bundle $\mathcal{E}$ on $\mathcal{X}_\mathfrak{o}$ extending E such that the pullback of the special fibre $\mathcal{E}_k$ of $\mathcal{E}$ to all normalized irreducible components of $\mathcal{X}_k$ is strongly semistable of degree zero. We denote the (full) category of all strongly semistable bundles on $X_{\mathbb{C}_p}$ by $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$.

Theorem 5 *For every vector bundle E on $X_{\mathbb{C}_p}$ with strongly semistable reduction of degree zero there is a divisor D such that E is contained in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$. In other words, we have $\mathfrak{B}^s_{X_{\mathbb{C}_p}} = \bigcup_D \mathfrak{B}_{X_{\mathbb{C}_p},D}$.*

For the **proof** see [De-We2], theorem 36. The idea is the following. We have just seen that all categories $\mathfrak{B}_{X_{\mathbb{C}_p},D}$ are contained in $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$. Hence it remains to show that for every bundle E on $X_{\mathbb{C}_p}$ with strongly semistable reduction $\mathcal{E}_k$ of degree zero there is a divisor D on X such that E is contained in $\mathfrak{B}_{X_{\mathbb{C}_p},D}$. There is a finite extension K of $\mathbb{Q}_p$ such that the model $\mathcal{X}$ descends to a model $\mathcal{X}_{\mathfrak{o}_K}$ over the ring of integers in K and such that the special fibre $\mathcal{E}_k$ of $\mathcal{E}$ descends to a vector bundle $\mathcal{E}_{\mathbb{F}_q}$ on the special fibre $\mathcal{X}_{\mathbb{F}_q}$ of $\mathcal{X}_{\mathfrak{o}_K}$. By enlarging K if necessary, we can assume that the irreducible components $D_1, \dots, D_r$ of $\mathcal{X}_{\mathbb{F}_q}$ are geometrically irreducible, hence they give the components $C_1, \dots, C_r$ of $\mathcal{X}_k$ by base change. Then the pullback of $\mathcal{E}_{\mathbb{F}_q}$ to all normalized components $\tilde{D}_\nu$ is strongly semistable of degree zero.

Now on the smooth, projective curve $\tilde{D}_\nu$ over the finite field $\mathbb{F}_q$ there are only finitely many isomorphism classes of semistable bundles of degree zero. This implies that on $\mathcal{X}_{\mathbb{F}_q}$ there are only finitely many isomorphism classes of bundles whose pullbacks to all normalized components $\tilde{D}_\nu$ are semistable of degree zero (see the proof of [De-We2], theorem 18). For $q = p^r$ we put $F_q = F^r$, i.e. F_q is the Frobenius fixing $\mathbb{F}_q$. We regard F_q also as a $\mathbb{F}_q$-linear automorphism of $\mathcal{X}_{\mathbb{F}_q}$ and consider all pullbacks $F_q^{k*}\mathcal{E}_{\mathbb{F}_q}$. This gives an infinite collection of bundles whose pullback to all $\tilde{D}_\nu$ is semistable of degree zero. Since there are only finitely many isomorphism classes available, we find two bundles in this collection which are isomorphic. Hence there are natural numbers $r > s$ satisfying $F_q^{r*}\mathcal{E}_{\mathbb{F}_q} \simeq F_q^{s*}\mathcal{E}_{\mathbb{F}_q}$, i.e. for $n = r - s$ we find $F_q^{n*}(F_q^{s*}\mathcal{E}_{\mathbb{F}_q}) \simeq F_q^{s*}\mathcal{E}_{\mathbb{F}_q}$. By the theorem of Lange and Stuhler cited in section 4, this implies that $F_q^{s*}\mathcal{E}_{\mathbb{F}_q}$ is a finite bundle, i.e. it becomes trivial on a finite étale covering $\omega : Y_0 \to \mathcal{X}_{\mathbb{F}_q}$. Hence $\mathcal{E}_{\mathbb{F}_q}$ becomes trivial after pullback via $\omega \circ F_q^s$.

Now we have to lift this covering of the special fibre $\mathcal{X}_{\mathbb{F}_q}$ to a covering of the whole model $\mathcal{X}_{\mathfrak{o}_K}$. This is no problem for the étale part ω, which lifts to a finite, étale morphism $\omega_{\mathfrak{o}_K} : \tilde{\mathcal{Y}}_{\mathfrak{o}_K} \to \mathcal{X}_{\mathfrak{o}_K}$, but a non-trivial task for the Frobenius. As a first step we find a semistable, regular and projective $\mathfrak{o}_K$-scheme $\mathcal{Y}_{\mathfrak{o}_K}$ together with a morphism $\mathcal{Y}_{\mathfrak{o}_K} \to \tilde{\mathcal{Y}}_{\mathfrak{o}_K}$ (we might have to enlarge K here). The map $\theta : \mathcal{Y}_{\mathfrak{o}_K} \to \tilde{\mathcal{Y}}_{\mathfrak{o}_K} \to \mathcal{X}_{\mathfrak{o}_K}$ also has the property that $\theta_{\mathbb{F}_q} \circ F_q^s$ trivializes the special fibre of $\mathcal{E}_{\mathbb{F}_q}$ by pullback.

We embed $\mathcal{Y}_{\mathfrak{o}_K}$ in some projective space $\mathbb{P}^N_{\mathfrak{o}_K}$. Now we define a Frobenius lift f_{q^s} on $\mathbb{P}^N_{\mathfrak{o}_K}$ by $[x_0, \dots, x_N] \mapsto [x_0^{q^s}, \dots, x_N^{q^s}]$ on projective coordinates. The special fibre of f_{q^s} is the Frobenius F_q^s. Let $\mathcal{Y}'_{\mathfrak{o}_K}$ be the base change of $\mathcal{Y}_{\mathfrak{o}_K}$

by f_{q^s}, i.e. $\mathcal{Y}'_{\mathfrak{o}_K}$ sits in a cartesian diagram

$$\begin{array}{ccc} \mathcal{Y}'_{\mathfrak{o}_K} & \longrightarrow & \mathcal{Y}_{\mathfrak{o}_K} \\ \big\downarrow & & \big\downarrow \tau \\ \mathbb{P}^N_{\mathfrak{o}_K} & \xrightarrow{f_{q^s}} & \mathbb{P}^N_{\mathfrak{o}_K} \end{array}$$

Of course, the generic fibre of this Frobenius lift f_{q^s} is étale only outside the union of the coordinate hyperplanes. But we can twist the projective embedding τ of $\mathcal{Y}_{\mathfrak{o}_K}$ into $\mathbb{P}^N_{\mathfrak{o}_K}$ by an automorphism in PGL_N so that the morphism $\mathcal{Y}'_{\mathfrak{o}_K} \to \mathcal{Y}_{\mathfrak{o}_K} \to \mathcal{X}_{\mathfrak{o}_K}$ is generically étale outside a divisor D on the generic fibre X_K of $\mathcal{X}_{\mathfrak{o}_K}$.

Now we look at the special fibre of this morphism $\mathcal{Y}'_{\mathfrak{o}_K} \to \mathcal{Y}_{\mathfrak{o}_K} \to \mathcal{X}_{\mathfrak{o}_K}$. There is a natural map $i = (\tau_{\mathbb{F}_q}, F_q^s) : \mathcal{Y}_{\mathbb{F}_q} \to \mathcal{Y}'_{\mathbb{F}_q}$. By definition, it gives the Frobenius F_q^s after composition with the projection to $\mathcal{Y}_{\mathbb{F}_q}$. Since $\mathcal{Y}_{\mathbb{F}_q}$ is semistable, it is reduced. Besides, i induces an isomorphism with the reduced induced structure of $\mathcal{Y}'_{\mathbb{F}_q}$, see [De-We2], lemma 19. Now we dominate $\mathcal{Y}'_{\mathfrak{o}_K}$ by a semistable $\mathfrak{o}_K$-scheme $\mathcal{Z}_{\mathfrak{o}_K}$ (possibly after enlarging K). Thus we get a chain of morphisms $\mathcal{Z}_{\mathfrak{o}_K} \to \mathcal{Y}'_{\mathfrak{o}_K} \to \mathcal{Y}_{\mathfrak{o}_K} \to \mathcal{X}_{\mathfrak{o}_K}$. Let us look at the special fibres

$$\mathcal{Z}_{\mathbb{F}_q} \to \mathcal{Y}'_{\mathbb{F}_q} \to \mathcal{Y}_{\mathbb{F}_q} \to \mathcal{X}_{\mathbb{F}_q}.$$

Since the semistable curve $\mathcal{Z}_{\mathbb{F}_q}$ is reduced, the first map factors through the reduced induced structure of $\mathcal{Y}'_{\mathbb{F}_q}$, hence through i. But i composed with the projection $\mathcal{Y}'_{\mathbb{F}_q} \to \mathcal{Y}_{\mathbb{F}_q}$ is the Frobenius F_q^s. Hence $\mathcal{Z}_{\mathbb{F}_q} \to \mathcal{X}_{\mathbb{F}_q}$ factors through $\theta_{\mathbb{F}_q} \circ F_q^s$ which is equal to $F_q^s \circ \theta_{\mathbb{F}_q}$. Hence the pullback of $\mathcal{E}_{\mathbb{F}_q}$ is trivial.

Going up to $\mathfrak{o}$, we get a finitely presented, proper morphism $\pi : \mathcal{Z} \to \mathcal{X}$ which generically is finite and étale over $X \backslash D$ such that the special fibre of $\pi_{\mathfrak{o}}^* \mathcal{E}$ is trivial. Hence E lies in $\mathfrak{B}_{X_{\mathbb{C}_p}, D}$ by theorem 3, which proves our claim. □

In this argument, the freedom of changing the embedding of $\mathcal{Y}_{\mathfrak{o}_K}$ into projective space by an automorphism in PGL_N can be further exploited to find a second divisor D' on X which is disjoint from D such that E also lies in $\mathfrak{B}_{X_{\mathbb{C}_p}, D'}$. Hence by the construction in section 2, we can define parallel transport on E along étale paths in $X \backslash D$ and along étale paths in $X \backslash D'$. It can be shown that these two constructions fit together on the intersection of the open subcurves $X \backslash D$ and $X \backslash D'$ and hence give rise to parallel transport along étale paths in the whole of X (see [De-We2], proposition 34). Applying theorem 2 we then deduce the following corollary (see [De-We2], theorem 36).

Corollary 6 *There is a functor ρ from $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$ to the category of representations of $\Pi_1(X)$ on finite-dimensional $\mathbb{C}_p$-vector spaces, which is exact and commutes with tensor products, duals, internal homs and exterior powers. Besides, it behaves fonctorially with respect to pullbacks along morphisms of curves over $\overline{\mathbb{Q}}_p$ and is compatible with Galois-conjugation.*

Let $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ be the (full) category of all vector bundles on $X_{\mathbb{C}_p}$ for which there exists a finite étale covering $\alpha_{\overline{\mathbb{Q}}_p} : Y \to X$ over $\overline{\mathbb{Q}}_p$ such that α^*E has strongly semistable reduction of degree zero on $Y_{\mathbb{C}_p}$, where $\alpha : Y_{\mathbb{C}_p} \to X_{\mathbb{C}_p}$ is the base change to $\mathbb{C}_p$. If E lies in $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$, we say that E has potentially strongly semistable reduction of degree zero.

Let E be a bundle on $X_{\mathbb{C}_p}$ with potentially strongly semistable reduction of degree zero, and let $\alpha_{\overline{\mathbb{Q}}_p} : Y \to X$ be a finite étale covering as above. We can assume that $Y \to X$ is a Galois covering. By corollary 6, α^*E defines a representation of the fundamental groupoid $\Pi_1(Y)$. Let γ be an étale path from x to x' on X. Once we choose a point y in Y lying over x, the path γ can be lifted to an étale path δ on Y from y to some point y' over x'. Then we define

$$\rho_E(\gamma) = \rho_{\alpha^*E}(\delta) : E_x = (\alpha^*E)_y \to (\alpha^*E)_{y'} = E_{x'}.$$

Choosing another lift δ amounts to choosing another point $\tilde{y}$ over x as starting point. Hence there is a Galois automorphism $\sigma \in \mathrm{Gal}(Y/X)$ with $\sigma(y) = \tilde{y}$. If $\sigma_* : \Pi_1(Y) \to \Pi_1(Y)$ denotes the natural functor given by σ, then the path $\sigma_*\delta$ is the lift of γ with starting point $\sigma(y) = \tilde{y}$. Since $\sigma^*\alpha^*E = \alpha^*E$, it follows from corollary 6 that $\rho_{\alpha^*E}(\delta) = \rho_{\alpha^*E}(\sigma_*\delta)$. Hence ρ_E is well-defined.

Therefore the construction of representations of the fundamental groupoid extends to bundles in $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$. All the properties stated in corollary 6 also hold for bundles in $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$.

12. How big are our categories of bundles? It is easy to see that all the categories $\mathfrak{B}_{X_{\mathbb{C}_p},D}$, $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$ and $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ are closed under direct sums, tensor products, internal homs and exterior powers. The following theorem collects more information about these categories.

Theorem 7 *i) The category $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ contains all line bundles of degree zero.*

ii) For every divisor D, the categories $\mathfrak{B}_{X_{\mathbb{C}_p},D}$, $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$ and $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ are stable under extensions, i.e. if $0 \to E' \to E \to E'' \to 0$ is an exact sequence of vector bundles such that E' and E'' are in the respective category, the same holds for E.

iii) If E is contained in $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$, respectively $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$, then every subbundle of degree zero and every quotient bundle of degree zero is also contained in $\mathfrak{B}^s_{X_{\mathbb{C}_p}}$, respectively $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$.

A **proof** for i) can be found in [De-We2], theorem 12, ii) is proven in [De-We2], theorem 11, and iii) is shown in [De-We3], theorem 9.

Theorem 7,iii) implies that $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ together with its natural fibre functor $E \mapsto E_x$ for some fixed $x \in X(\mathbb{C}_p)$ is a neutral Tannaka category, see [De-We3], theorem 12.

If X is an elliptic curve, then by Atiyah's classification [At] of vector bundles on $X_{\mathbb{C}_p}$ the category $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ contains all semistable bundles of degree zero, cf. [De-We2], corollary 15. Hence for elliptic curves $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ coincide with the category of all semistable bundles of degree zero.

For curves of higher genus it is an open question, if the category $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ coincides with the category of all semistable bundles of degree zero on $X_{\mathbb{C}_p}$.

In [De-We3], theorem 12, it is shown that this is the case if and only if the corresponding subcategories of polystable bundles of degree zero coincide. Here a vector bundle on $X_{\mathbb{C}_p}$ is called polystable of degree zero if it is isomorphic to a direct sum of stable bundles of degree zero. Denote by $\mathfrak{T}^{\mathrm{ss}}_{\mathrm{red}}$ (respectively $\mathfrak{B}^{\mathrm{ps}}_{\mathrm{red}}$) the category of all polystable bundles of degree zero on $X_{\mathbb{C}_p}$ (repectively of all polystable bundles of degree zero contained in $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$). In [De-We3], theorem 16, it is shown that the Tannaka groups of $\mathfrak{T}^{\mathrm{ss}}_{\mathrm{red}}$ and $\mathfrak{B}^{\mathrm{ps}}_{\mathrm{red}}$ have the same group of connected components.

13. Representations of the fundamental group. If we fix a base point $x \in X(\mathbb{C}_p)$ and restrict the functor ρ to closed étale paths in x, then every vector bundle E in $\mathfrak{B}^{ps}_{X_{\mathbb{C}_p}}$ gives rise to a continuous representation

$$\pi_1(X, x) \to \mathrm{Aut}(E_x)$$

of the fundamental group. In [De-We1], we define an analogue of $\mathfrak{B}_{X_{\mathbb{C}_p}}$ and ρ on an Abelian variety A over $\overline{\mathbb{Q}}_p$ with good reduction. We also show how our construction generalizes a map in [Ta] and how it is related to the Hodge-Tate decomposition of $H^1_{et}(A, \mathbb{Q}_p) \otimes \mathbb{C}_p$.

14. Mumford curves. Finally, we consider the special case that X is a Mumford curve (see [Mu]), i.e. X descends to a curve X_K over a finite extension K of $\mathbb{Q}_p$ which has a rigid analytic uniformization as Ω/Γ. Here Γ is a Schottky group in $\mathrm{PGL}(2, K)$ and Ω is the open subset of the rigid analytic $\mathbb{P}^1_K$ where Γ acts discontinously. Let $\mathcal{X}_{\mathfrak{o}_K}$ be the minimal regular model of X_K, see e.g. [Liu], section 9.3. Then all normalized irreducible components in the special fibre of $\mathcal{X}_{\mathfrak{o}_K}$ are $\mathbb{P}^1$'s.

In [Fa2], Faltings shows that there is an equivalence of categories between semistable vector bundles E of degree zero on X_K and K-linear representations of the Schottky group Γ which satisfy a certain boundedness condition. In [Re-Pu], his results were generalized to non-discrete p-adic base fields, e.g. $\mathbb{C}_p$. In [He], Herz shows how Faltings' construction is related to the representations of the fundamental group given by bundles in $\mathfrak{B}_{X_{\mathbb{C}_p}}$.

Namely, let E be a vector bundle on $X_{\mathbb{C}_p}$ which can be extended to a vector bundle $\mathcal{E}$ on the minimal model $\mathcal{X}_{\mathfrak{o}}$. By the generalization [Re-Pu] of Faltings' results, E gives rise to a representation π of the Schottky group Γ. Herz shows that this representation comes in fact from a represention (also called π) of Γ on a free $\mathfrak{o}$-module. Moreover, he shows that E lies in $\mathfrak{B}_{X_{\mathbb{C}_p}}$. Let x be a base point in $X(\mathbb{C}_p)$ and $x_{\mathfrak{o}}$ in $\mathcal{X}_{\mathfrak{o}}(\mathfrak{o})$ its extension. By construction, the representation $\rho_E : \pi_1(X, x) \to \mathrm{Aut}(E_x)$ is induced from a representation

$$\rho_{\mathcal{E}} : \pi_1(X, x) \to \mathrm{Aut}(\mathcal{E}_{x_{\mathfrak{o}}})$$

on the $\mathfrak{o}$-lattice $\mathcal{E}_{x_{\mathfrak{o}}}$ in E_x, where $\rho_{\mathcal{E}}$ is the limit of representations $\rho_{\mathcal{E},n} : \pi_1(X, x) \to \mathrm{Aut}(\mathcal{E}_{x_n})$. It is shown in [He] that $\rho_{\mathcal{E},n}$ factors through a finite quotient of Γ for a suitably chosen model $\mathcal{E}$, and that the induced representation on this quotient is isomorphic to the reduction of π modulo p^n. Hence $\rho_{\mathcal{E}}$ factors through the profinite completion $\widehat{\Gamma}$ of the Schottky group Γ and is isomorphic to the profinite completion of Faltings' representation π.

References

[At] M. Atiyah: Vector bundles over an elliptic curve. Proc. London Math. Soc. **7** (1957), 414–452

[De-We1] C. Deninger, A. Werner: Line bundles and p-adic characters. In: G van der Geer, B. Moonen, R. Schoof (eds): Number Fields and Function Fields - Two Parallel Worlds. Birkhäuser Progress in Mathematics, Vol 239, 2005, 101-131.

[De-We2] C. Deninger, A. Werner: Vector bundles on p-adic curves and parallel transport. Ann. Scient. Éc. Norm. Sup. **38** (2005) 553-597.

[De-We3] C. Deninger, A. Werner: On Tannaka duality for vector bundles on p-adic curves. Preprint unter http://arxiv.org/abs/math/0506263.

[Fa1] G. Faltings: Semistable vector bundles on Mumford curves. Invent. Math. **74** (1983), 199–212

[Fa2] G. Faltings: A p-adic Simpson correspondence. Adv. Math. **198** (2005) 847-862.

[He] G. Herz: On representations attached to semistable vector bundles on Mumford curves. Thesis Münster 2005.

[La-Stu] H. Lange, U. Stuhler: Vektorbündel auf Kurven und Darstellungen der algebraischen Fundamentalgruppe. Math. Z. **156** (1977), 73–83

[Liu] Q. Liu: Algebraic Geometry and Arithmetic Curves. Oxford University Press 2002

[Mu] D. Mumford: An analytic construction of degenerating curves over complete local rings. Compositio Math. **24** (1972) 129-174.

[Na-Se] M.S. Narasimhan, C.S. Seshadri: Stable and unitary vector bundles on a compact Riemann surface. Ann. Math. **82** (1965), 540–567

[Re-Pu] M. Reversat, M. van der Put: Fibrés vectoriels semi-stables sur une courbe de Mumfort. Math. Ann. **273** (1986) 573-600.

[Ta] J. Tate: p-divisible groups. In: Proceedings of a Conference on local fields. Driebergen 1966, 158–183

[Weil] A. Weil: Généralisation des fonctions abéliennes. J. de Math. P. et App. (IX) 17 (1938) 47-87

Author's address: Department of Mathematics, Universität Stuttgart, Pfaffenwaldring 57, D - 70569 Stuttgart
email: werner@mathematik.uni-stuttgart.de