CONSTRUCTION OF SOLUTIONS TO SERRIN'S OVERDETERMINED PROBLEM ON THE 2-SPHERE

IGNACE ARISTID MINLEND

ABSTRACT. A celebrated result of Serrin, with applications in fluid dynamics, states that if $\Omega \subset \mathbb{R}^N$ is a C^2 regular bounded domain in which the overdetermined problem

$$\begin{aligned} -\Delta u &= 1 && \text{in } \Omega \\ u &= 0, \quad \frac{\partial u}{\partial \eta} = c && \text{on } \partial\Omega \end{aligned} \tag{0.1}$$

has a solution, then Ω is a ball. Here $c > 0$ is a positive constant, and η is the unit outer normal vector field on $\partial\Omega$. In this talk we present a construction in the Euclidean sphere $\mathbb{S}^2$ of a continuous family of regular and bounded domains $(\Omega_s)_{s\in(-\varepsilon_0,\varepsilon_0)}$ in which Serrin's overdetermined problem (0.1) is solvable. This family of domains bifurcates from a closed band in $\mathbb{S}^2$ which is symmetric with respect to the equator, and it is obtained via the Crandall-Rabinowitz Bifurcation Theorem.