EXISTENCE OF SELF-CHEEGER SETS ON RIEMANNIAN MANIFOLDS

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ABSTRACT. Let Ω be an open set with smooth boundary in a Riemannian Riemannian manifold $(\mathcal{M}, g)$. The Cheeger constant $h(\Omega)$ of Ω is defined by

$$h(\Omega) := \inf_{A} \frac{P(A)}{|A|}, \tag{0.1}$$

where the domain A varies over all measurable subsets of Ω with finite perimeter $P(A)$, and $|A|$ is the N-dimensional Riemannian volume of A.

Any set F in Ω which realizes the infimum (0.1) is called a Cheeger set in Ω. If Ω is it self a minimizer for (0.1), we say that Ω *is self-Cheeger*. If Ω is the only set which attains $h(\Omega)$, we say that Ω is *uniquely self-Cheeger*.

In this this talk we prove existence in any compact Riemannian manifold $(\mathcal{M}, g)$ of dimension $N \geq 2$, of a family of *uniquely* self-Cheeger sets $(\Omega_\varepsilon)_{\varepsilon \in (0,\varepsilon_0)}$ with

$$h(\Omega_\varepsilon) = \frac{N}{\varepsilon}.$$

The domains Ω_ε are perturbations of geodesic balls of radius ε centered at $p \in \mathcal{M}$, and in particular, if p_0 is a non-degenerate critical point of the scalar curvature of g, then the family $(\partial\Omega_\varepsilon)_{\varepsilon \in (0,\varepsilon_0)}$ constitutes a smooth foliation of a neighborhood of p_0.