HARDY-SOBOLEV INEQUALITY WITH CYLINDRICAL WEIGHT ON RIEMANNIAN MANIFOLDS

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ABSTRACT. Let $(\mathcal{M}, g)$ be a smooth compact Riemannian manifold of dimension $N \geq 3$ and we let Σ a closed minimal submanifold of dimension $1 \leq k \leq N-2$. We denote by $\rho(p) = \text{dist}_g(p, \Sigma)$ the geodesic distance from $p \in \mathcal{M}$ to Σ. Given $\sigma \in (0, 2)$, we study existence of positive distributional solution $u \in H^1(\mathcal{M})$ to the critical equation

$$\Delta_g u + hu = \rho^{-\sigma} u^{2^*(\sigma)-1} \quad \text{in} \quad \mathcal{M} \tag{0.1}$$

where $\Delta_g := -\text{div}_g(\nabla)$ is the Laplace Beltrami operator, $2^*(\sigma) := \frac{2(N-\sigma)}{N-2}$ is the critical Hardy-Sobolev exponent and $h \in C^0(\mathcal{M})$. In particular via minimization method in the spirit of Aubin, we prove existence of minimizers under the influence of the scalar curvature, the second fundamental form and the potential h.

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