Real-valued valuations defined on the space of quasi-concave functions

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We are going to present results from my PhD thesis concerning the study of valuations on the space of quasi-concave functions.

Valuations Theory arose from the resolution of the third Hilbert problem and it was basically introduced for the space of convex bodies, $\mathcal{K}^n$, i.e. compact and convex subsets of $\mathbb{R}^n$.
We are going to present valuations defined on function space. Let X be a generic space of real-valued functions defined on $\mathbb{R}^n$, then a real-valued valuations is a functional $\mu\colon X \to \mathbb{R}$ with the following property:

$$\mu(f) + \mu(g) = \mu(f \vee g) + \mu(f \wedge g),$$

for all $f, g \in X$ such that $f \vee g$ and $f \wedge g \in X$, where

$$f \vee g(x) = \max\{f(x), g(x)\}, \quad f \wedge g(x) = \min\{f(x), g(x)\}$$

for all $x \in \mathbb{R}^n$.

We studied the space of quasi-concave functions, i.e. $f\colon \mathbb{R}^n \to \mathbb{R}_+$ such that, for all $t > 0$,

$$L_t(f) = \{x \in \mathbb{R}^n | \, f(x) \geq t\} \in \mathcal{K}^n \cup \{\varnothing\}.$$

We are going to present some basic results we obtained, for example characterization results of continuous, with respect to appropriate convergences, and invariant, with respect to several groups that act on $\mathbb{R}^n$, valuations, focusing on the connection with $\mathcal{K}^n$.