A finite element method for the simulation of motility of living cells

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N. Sfakianakis has been supported during the development of this work by the "*Alexander von Humboldt Foundation*" and the "*Center for Computational Sciences in Mainz*".

The cytoskeleton is a cellular skeleton inside the cytoplasm of living cells. The front of the cytoskeleton, also known as lamellipodium, is the driving mechanism of the cells motility. The lamellipodium is comprised by long double helix chains of actin protein termed actin-filaments.

The filaments have one of their ends at the cell membrane (plus end), and their other end (minus end) inside the cytoplasm. They polymerize by addition of actin monomers in their plus end and at the same time they depolymerize at their minus end. They are inextensible, behave like elastic beams with friction to the substrate. They exhibit crosslinks between each other and are subjects of forces exerted on them by the membrane as well as contractile forces caused by myosin. Moreover, new filaments are nucleated at the membrane.

We develop in this work ([3])a Finite Element method for the simulation of the both stationary -not moving but still highly dynamic structures- as well as moving lamellipodia.

The model we resolve numerically was proposed in [1, 2] and is 4th order parabolic delay problem. It assumes that the lammelipodium is comprised of two families of 2 dimensional filaments. The *System* that stems reads as:

$$\underbrace{\mu^B \partial_s^2 \left(\eta^\pm \partial_s^2 F^\pm\right)}_{\text{bending}} - \underbrace{\partial_s \left(\eta^\pm \lambda^\pm \partial_s F^\pm\right)}_{\text{in-extensibility}} + \underbrace{\eta^\pm \mu^A D_t^\pm F^\pm}_{\text{adhesion}}$$
$$\pm \underbrace{\partial_s \left(\eta^+ \eta^- \mu_\pm^T (\varphi - \varphi_0) \partial_s F^{\pm\perp}\right)}_{\text{twisting}} \pm \underbrace{\eta^+ \eta^- \mu^S \left(D_t^+ F^+ - D_t^- F^-\right)}_{\text{stretching}} = 0$$

where with $\pm$ we denote the two families of filaments, $F^\pm(s, \alpha, t) \in \mathbb{R}^2$, $s \in [0, L]$, $\alpha \in [0, 2\pi]$, $t > 0$ describes the position of the filament α of the family $\pm$ at time t, L is the maximal length of the filaments, $\eta^\pm$ are distributions functions that for the graded length, ϕ_0 is the preferred angle of the crosslinked filaments, ϕ their actual angle, and μ^B, μ^A, μ^T, μ^S are the state parameters of the problem. $D_t = \partial_t - v\partial_s$ is the material derivative operator where v is the polymerization speed.

The discretization of the *System* is with respect to (s, a) and $t > 0$. We have used two dimensional finite element method with Hermite basis function along the s-direction, and Lagrange basis along the a-direction.

The non-linearity in the in-extensibility term is treated by an implicit-explicit discretization; this gives rise to two more equations for $\lambda\pm$. The adhesion term is discretized explicitly in time. The stretching and twisting terms couple the two families; the temporal derivatives in the stretching term are treated by a predictor-corrector step.

The innovation of this work resides in both the use of non standard finite elements, as well as in the fact that we provide in this work e.g. [3] the numerical simulations of moving cells while bridging the passing from the micro- to the meso-scale.

References

[1] Oelz, D. and Schmeiser, Ch. and Small, J.V., Modelling of the Actin-cytoskeleton in symmetric lamellipodial fragments, Cell Adhesion and Migration, 2008

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[3] Manhart, A. and Oelz D. and Schmeiser, Ch. and Sfakianakis, N. A Finite Element method for the simulation of the lamellipodium of living cells, (preprint) 2014

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