On Riemann's existence theorem for arithmetic Galois groups
(joint work with Kay Wingberg)

If k is a number field and $k(p)$ denotes its maximal p-extension for a prime number p, then the following result can be considered as a number theoretic analogue of Riemann's existence theorem for Riemann surfaces: The inertia groups of a family of primes of k generate a free pro-p-product inside the Galois group $G(k(p)|k)$. More precisely, if S is a (possibly infinite) set of primes of k containing the primes above p and the infinite primes, the canonical homomorphism

$$\mathop{\ast}_{\mathfrak{p} \notin S(k_S(p))} T(k_{\mathfrak{p}}(p)|k_{\mathfrak{p}}) \longrightarrow G(k(p)|k_S(p))$$

is bijective. Here a generalized notion of free products has to be used taking into account that the inertia groups $T(k_{\mathfrak{p}}(p)|k_{\mathfrak{p}})$ vary continuously with respect to a profinite topology on $\mathrm{Spec}(\mathcal{O}_{k_S(p)})$.

It is a natural question to ask if an analogous result holds for the decomposition groups of an infinite family of primes. We report on joint work with Kay Wingberg where the notion of free products of non-compact corestricted bundles of profinite groups is introduced and give their main properties generalizing results of Neukirch, Gildenhuys-Ribes, Melnikov et al. Using arithmetic results of Wingberg, we discuss an analogue of Riemann's existence theorem for so-called stably saturated sets of primes.