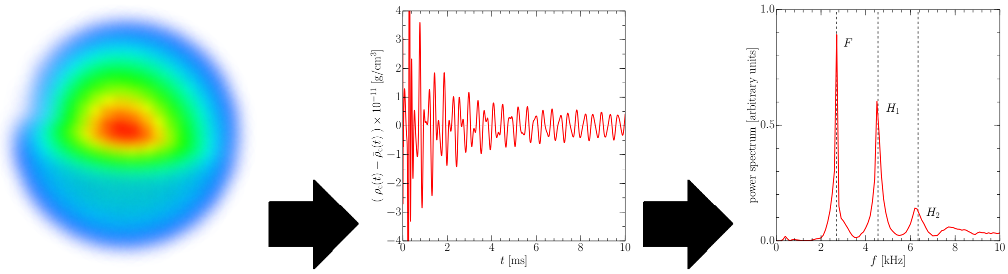

Experiments in Computer Simulations : Radial Oscillations of Relativistic Spherical Star

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The setup and the description of this experiment was done by *Prof. Dr. Kentaro Takami*. He is now teaching at the 'Kobe City College of Technology' in Japan.

0 Class Information

- application form :
  - if you want to do this experiment, please register via e-mail to Matthias Hanauske no later than on the last Wednesday before the week in which you want to do the experiment; your e-mail should include the following information: (1) student number, (2) full name, (3) e-mail address.
- intensive course :
  - 12 [hours] = 6 [hours/week] × 2 [weeks].
- when :
  - Monday 9-16, two subsequent weeks upon individual arrangement with M. Hanauske; other time slots may be arranged with M. Hanauske individually.
- where :
  - Pool Room 01.120 (or, if this room is not available during the time slot chosen, Pool Room _.501).
- preparation :
  - an account for you on the "FUCHS" cluster of the CSC (https://csc.uni-frankfurt.de/index.php?id=4 will be provided; please read the quick starting guide on the CSC web pages before starting the simulation.
- required skill :
  - basic Linux knowledge.
- using software :
  - `Einstein Toolkit` [1].
  - `gnuplot` (http://www.gnuplot.info/)

– `pygraph` (https://bitbucket.org/dradice/pygraph).
– `Grace` (http://plasma-gate.weizmann.ac.il/Grace/).
– `python` (https://www.python.org/) and `matplotlib` (http://matplotlib.org/).
– `Mathematica`

1 Introduction

Stellar Oscillations give us many information of a star, such as an internal structure. Therefore they are well studied from the both sides of theory and observation, especially for the Sun as a *"helioseismology"*. These studies basically can be adapted to all of Newtonian stars as a *"asteroseismology"*. In fact, the same treatment are used for the pulsations of e.g., the Cepheid variable, and the properties are accurately extracted.

Basically we can use same idea for relativistic stars such as a neutron star. However the basic equations are very complicate and it is very difficult to analytically consider them. In fact, we do not know any analytic treatment for a rotating full relativistic star. Thanks to recent development of a numerical relativity and computer fluid dynamics, fortunately we can tackle the problems by using computer simulations. Then e.g., Ref. [3, 4, 9, 7] have been obtained.

In this experiment class, we learn how to extract oscillations of a relativistic star via computer simulations. The flowchart is (i) a construction of a star and the evolution in a computer, (ii) extraction of oscillations, and (iii) identification of the eigenfrequencies (see figures in top of page 1). We concentrate only radial oscillations of a spherical relativistic star, i.e., Tolman-Oppenheimer-Volkoff (TOV) solution, although these can be analytically computed by using a linear perturbation theory [8].

Unless explicitly stated, we use units in which $c = G = M_\odot = 1$ in this document.

2 Simulation and Analysis

2.1 Initial Data of TOV Star

In a spherical symmetry condition, a metric can be written as

$$ds^2 = -e^{2\nu(r)}(cdt)^2 + e^{2\mu(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \ . \tag{1}$$

Then TOV equations,

$$\frac{dm}{dr} = \frac{4\pi \ e \ r^2}{c^2} \ , \tag{2}$$
$$\frac{d\nu}{dr} = \frac{Ge^{2\mu}}{c^4 r^2}\left(mc^2 + 4\pi p r^3\right) \ , \tag{3}$$
$$\frac{dp}{dr} = -(\ e + p\ )\ \frac{d\nu}{dr} \ , \tag{4}$$

are derived (e.g., see [5]) from the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \ , \tag{5}$$

a conservation of energy momentum, ${T^\nu}_{\mu;\nu} = 0$, and a stress-energy tensor for perfect fluid,

$$T_{\mu\nu} = \left(\rho + \frac{\rho\varepsilon}{c^2} + \frac{p}{c^2}\right)u_\mu u_\nu + p g_{\mu\nu} = \left(\frac{e+p}{c^2}\right)u_\mu u_\nu + p g_{\mu\nu} \ . \tag{6}$$

Therefore a TOV star with polytropic equation of state (EOS), $p = K\rho^\Gamma$, $\Gamma = 2$ and $K = 100$ in $c = G = M_\odot = 1$ units, is constructed by integrating the ordinary differential equations form a certain central density ρ_c.

2.2 Time Evolution of Initial Data

The TOV star is perturbed by simply a numerical noise in this experiment. Then the perturbed star is numerically evolved by solving the relativistic hydro equations under (a) the fixed spacetime (Cowling approximation), or (b) the dynamical spacetime in Baumgarte-Shapiro-Shibata-Nakamura-Oohara-Kojima (BSSNOK) formulation [6, 2] which is modification of the $3 + 1$ ADM formalism. In practice, for these purposes we use a publicly available `Einstein Toolkit` [1] on the CSC-FUCHS cluster (https://csc.uni-frankfurt.de/index.php?id=49).

2.3 Extraction of Radial Oscillations

We extract the radial oscillations from the time variation of the central density $\rho_c(t)$ as in a figure (ii) in top of page 1. Then the frequencies of a fundamental (F) and the overtone modes ($H_1, H_2, \cdots$) for the radial oscillations are identified via a discrete Fourier transformation (DFT) (see figure (iii) in top of page 1).

3 Tasks

3.1 Task 1: Set Up Laboratory

Laboratory in computer simulations is the inside of a computer, especially here we use the supercomputer CSC-FUCHS. In order to start experiments, you have to understand the system and set up your environment in laboratory:

- load modules you need.
- download and compile the code.
- prepare a job submission script.
- submit a test run (e.g., , wave toy), check the results, and play around.

3.2 Task 2: Simulation of Oscillated TOV Star

Compute time evolution of an oscillated TOV star with $\rho_c(t = 0) = 0.00128$ in Cowling approximation by a low grid resolution $dx = dy = dz = 0.4$:

- understand the units of the code, i.e., $c = G = M_\odot = 1$, and the relation to CGS units.
- understand the model, and prepare the parameter file.
- evolve the model until ~ 10[ms] by using 24 cores. It takes < 2min if your setup is fine.
- understand the outputted data files, and plot these data by using e.g., `gnuplot` or `pygraph`.

3.3 Task 3: Identification of Eigenfrequencies for Radial Oscillations

Compute Power spectral density (PSD) of the variation of the central density $\rho(t)$ which is obtained in Task 1, and then compare the frequencies with the exact eigenfrequencies ($F \approx 2.686$[kHz], $H_1 \approx 4.550$[kHz], $H_2 \approx 6.342$[kHz], $H_3 \approx 8.108$[kHz]) computed by a linear perturbation theory:

- extract the variation of the central density $\rho(t)$ from simulation data.
- compute the PSD and check your peak frequencies by using the exact eigenfrequencies.
- prepare a figure such as a figure (iii) in top of page 1.

3.4 Task 4: Dependency for Grid Resolutions

In above tasks, you had a simulation with low grid resolution $dx = dy = dz = 0.4$, and extracted the frequencies. Of course the results has numerical error proportion to the grid resolution. Therefore, please consider the case of middle ($dx = dy = dz = 0.3$) and high ($dx = dy = dz = 0.2$) grid resolutions:

- please do same things with low grid resolution case for middle and high resolution cases.
- compare the results for three resolutions (low, middle, high) and make clear the differences. For example, please plot $\rho_c(t)$ for three resolutions.

3.5 Task 5: Oscillated TOV Star in Full Spacetime Evolution

In above, you assumed the Cowling approximation in the simplicity and cheap numerical cost. In order to check whether the approximation is appropriate or not, please compute the same model without the Cowling approximation, i.e., in full general relativistic evolution:

- compute and extract the frequencies.
- compare the frequencies to the case with the Cowling approximation. Is there difference? Is the approximation appropriate for the radial oscillations?

4 Final Report

You have to submit the report within 4 weeks after performing the simulation:

- please use the units such as kHz, g/cm^3, ms in the report, although $c = G = M_\odot = 1$ units is used in the code.
- please summarize what you learn and the results you get in this experiment.
- please include figures you plotted in the tasks.
- please tell me the frequencies which you identified in Task 5. At least, F and H_1 frequencies can be extracted.

References

[1] `http://einsteintoolkit.org`.

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[9] B. Zink, O. Korobkin, E. Schnetter, and N. Stergioulas. Frequency band of the f-mode Chandrasekhar-Friedman-Schutz instability. *Phys. Rev. D*, 81(8):084055, April 2010.