SFB/TRR 49, project A8: Interacting magnons and critical behavior of bosons Principal investigator: Prof. Dr. Peter Kopietz

In the second funding period of this SFB/TRR we continue our work on interacting magnon gases and boson systems. Our research is divided into three subprojects:

(a) Non-equilibrium dynamics of magnons in thin film ferromagnets.

(b) Magnons in frustrated antiferromagnets.

(c) Interacting bosons in reduced dimensions.

Our theoretical investigations in subproject (a) are performed in close contact with the experimental project A7 (Hillebrands, Serha), where correlated states of magnons out of equilibrium are studied using the Brillouin light scattering spectroscopy and other methods. In project A7 thin films of the magnetic insulator yttrium-iron garnet (YIG) are exposed to external microwave radiation, which generates non-equilibrium states in the magnon gas whose time evolution is measured. To achieve a theoretical understanding of these experiments, we develop a non-equilibrium many-body theory for the magnon gas in YIG. Previous approaches based on the time-dependent Hartree-Fock approximation ("S-theory") are not always satisfactory because this approximation does not allow for a first-principles description of damping and dissipation effects. Moreover, for strong microwave-pumping or for high magnon densities non-perturbative effects such as parametric resonance are observed, which cannot be described within simple perturbative techniques. We have therefore generalized the so-called functional renormalization group (FRG) approach to interacting Bose systems out of equilibrium. In the work [1] we have developed the general framework of calculating the time-evolution of distribution functions of interacting bosons using the FRG method. In particular, the problem of finding a proper cutoff function for the non-equilibrium FRG which does not violate causality has been carefully discussed in this work. Although in that work we have tested the non-equilibrium FRG only for a specific exactly solvable toy model involving only a single bosonic mode, the insights gained from this toy model were crucial for the subsequent application of the non-equilibrium FRG to YIG in our later work [2], where we have further developed the non-equilibrium FRG approach of Ref. [1] to describe the thermalization of the magnon gas in YIG due to coupling to a phonon bath in the absence of external pumping. The system is described by the Hamiltonian

$$\mathcal{H} = \sum_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}} + \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} b^\dagger_{\boldsymbol{q}} b_{\boldsymbol{q}} + \frac{1}{\sqrt{V}} \sum_{\boldsymbol{q}} \gamma_{\boldsymbol{q}} \rho_{-\boldsymbol{q}} (b_{\boldsymbol{q}} + b^\dagger_{-\boldsymbol{q}}), \tag{1}$$

where $a^\dagger_{\boldsymbol{k}}$ creates a magnon with momentum $\boldsymbol{k}$ and energy $\epsilon_{\boldsymbol{k}}$, while $b^\dagger_{\boldsymbol{q}}$ creates an acoustic phonon with momentum $\boldsymbol{q}$ and energy $\omega_{\boldsymbol{q}} = c|\boldsymbol{q}|$. The magnon-phonon coupling has been assumed to be of the form $\gamma_{\boldsymbol{q}} \propto |\boldsymbol{q}|$, which is a consequence of the fact that density fluctuations associated with the creation of longitudinal phonons are proportional to the divergence of the corresponding displacement field. In our most recent work [3] we have investigated the magnon-phonon coupling in YIG more thoroughly and have derived a more accurate Hamiltonian describing the magnon-phonon interaction in YIG. We succeeded in Ref. [2] to find a numerically tractable trunction of the non-equilibrium FRG flow equations for the magnon distribution in YIG, which has enabled us to calculate relaxation of the the non-equilibrium time-evolution into thermal equilibrium,

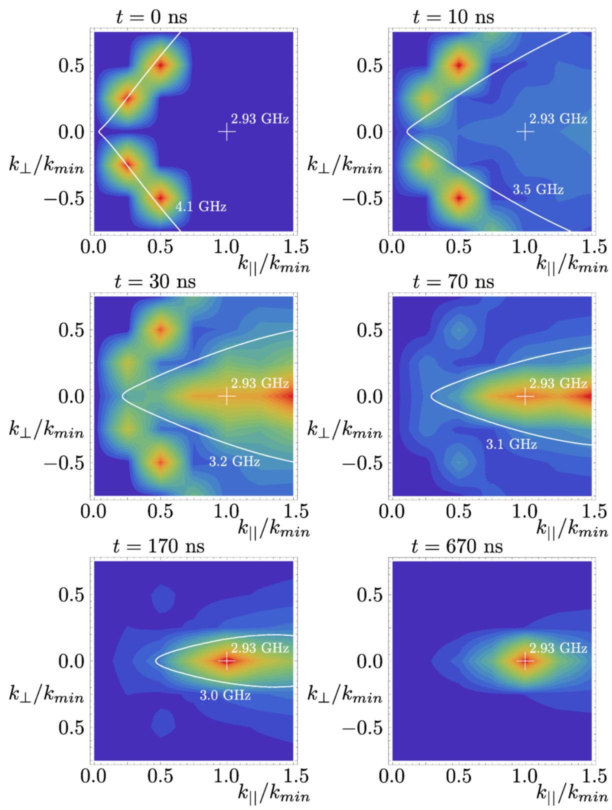

Figure 1: These contour plots show an interpolation of the magnon distribution of YIG on a 7×7-mesh in momentum space for different times. The white contour lines denote the magnon spectrum of YIG for fixed energies. The crosses mark the minimum of the dispersion. The results are presented such that they are directly comparable with the experimental data of Ref. [4]. We have shifted the zero of time to the moment where the microwave pumping is switched off. The initial state is chosen such that magnons with an energy of 4.1 GHz in the area $|k_\perp| < 0.5 k_{\mathrm{min}}$ are equally occupied, which resembles the initial magnon distribution in the experiment after the pumping has been switched off.

as shown in Fig. 1. These results agree quite well with the experimental results obtained in Ref. [4].

Subproject (b) is motivated by experimental investigations of project B1 (Wolf/Lang), where ultrasound propagation and thermodynamic properties of frustrated antiferromagnets (such as Cs_2CuCl_4) are studied in strong magnetic fields and at low temperatures. Depending on the orientation of an external magnetic field, the compound Cs_2CuCl_4 under investigation has a rather complex phase diagram, which includes the so-called cone-state exhibiting long-range magnetic order, and a magnetically disordered spin-liquid phase, as shown in Fig. 2. In the second funding period, we shall try to gain a better theoretical understanding of ultrasound experiments in the spin-liquid phase. Recently, we have shown (see the right panel of Fig. 2) that the ultrasound experiments probing the sound propagation aloung the direction of the strongest bond (the crystallographic b-axis) can be explained in wide interval of magnetic fields within the framework of a nearest neighbor spin-1/2 Heisenberg chain, where the spins are fermionized via a Jordan-Wigner transformation and the spin-phonon interaction arises from the usual exchange-striction mechanism. The dimensional reduction is also supported by our

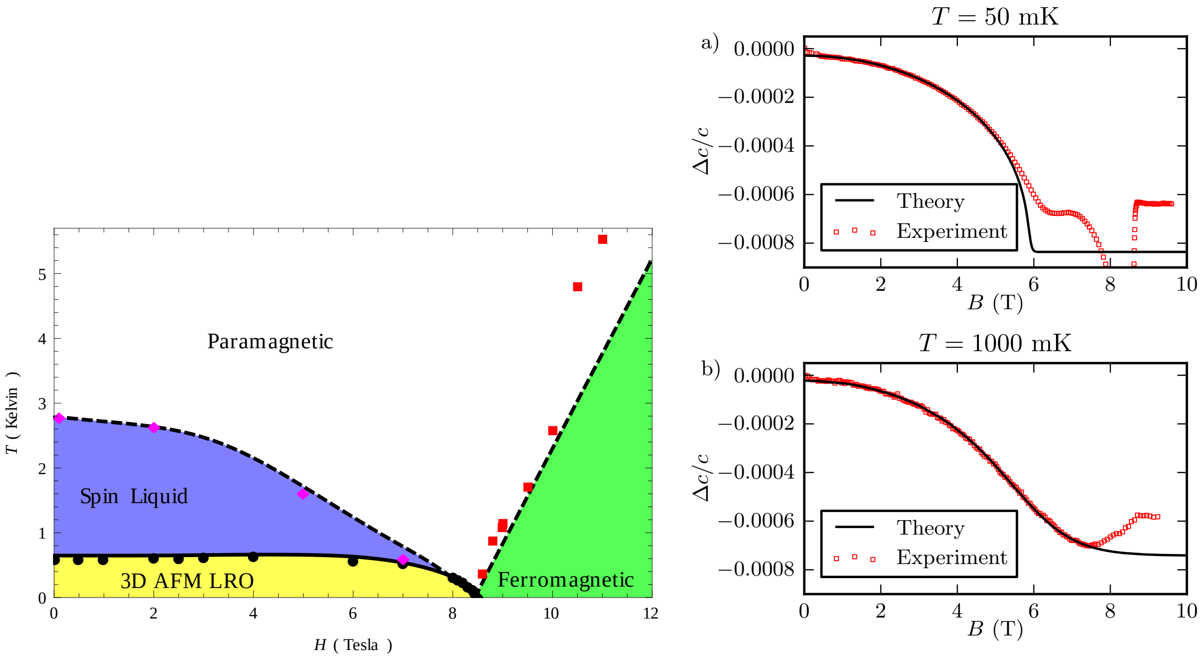

Figure 2: **Left panel:** Schematic phase diagram of Cs_2CuCl_4 as a function of temperature and a magnetic field. The dots dots represent experimental data. **Right panel:** Comparison of theory and experiment for the relative change of the sound velocity of the c_{22}-mode in Cs_2CuCl_4: a) in the ordered phase ($T = 50\,\mathrm{mK}$) b) in the spin-liquid phase ($T = 1\,\mathrm{K}$).

recent Majorana mean-field theory [6].

Finally, in subproject (c) we develop methods to calculate correlation functions of interacting bosons in reduced dimensions. In the work [5] we have used Popov's phase formalism to calculate the spectral line-shape and the damping of phase fluctuations of the one-dimensional interacting Bose gas at long wavelengths. Our long term goal it to develop FRG methods to calculate correlation functions of interacting bosons. For example, the dynamic structure factor of interacting bosons in one spatial dimension is known to exhibit a highly non-trivial line-shape which is characterized by algebraic threshold singularities. The line-shape close to the singularities is known, but a unified description of the line-shape for all energies does not exist.

References

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