Robust Hamiltonicity of random directed graphs

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Abstract

In his seminal paper from 1952 Dirac showed that the complete graph on $n \geq 3$ vertices remains Hamiltonian even if we allow an adversary to remove $\text{floor}(n/2)$ edges touching each vertex. In 1960, Ghouila-Houri obtained an analogue statement for digraphs by showing that every directed graph on $n \geq 3$ vertices with minimum in- and out-degree at least $n/2$ contains a directed Hamilton cycle. Both statements quantify the robustness of complete graphs (digraphs) with respect to the property of containing a Hamilton cycle.

A natural way to generalize such results to arbitrary graphs (digraphs) is using the notion of local resilience. The local resilience of a graph (digraph) $G$ with respect to a property $P$ is the maximum number $r$ such that $G$ has the property $P$ even if we allow an adversary to remove an $r$-fraction of (in- and out-going) edges touching each vertex. The theorems of Dirac and Ghouila-Houri state that the local resilience of the complete graph and digraph with respect to Hamiltonicity is $1/2$. Recently, this statements have been generalized to random settings. Lee and Sudakov (2012) proved that the local resilience of a random graph with edge probability $p=\omega(\log n / n)$ with respect to Hamiltonicity is $1/2 - o(1)$. For random directed graphs, Hefetz, Steger and Sudakov (2014+) proved an analogue statement, but only for edge probability $p=\omega(\log n / \sqrt{n})$. In this talk we present an improvement of their result to $p=\omega(\log^8 n / n)$, which is optimal up to the polylogarithmic factor.

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