On the Chromatic Number of Random Graphs

The interest in the chromatic number of the sparse random graph $G(n, p)$ dates back to the 1960 paper of Erdos and Renyi. In this talk I am going to present some recent progress on this question. Namely, I am going to show that for fixed k, random graphs with average degree

$$np \quad = \quad 2k \ln(k) - \ln(k) - 2\ln(2) - o(1) \qquad (*)$$

are k-colorable w.h.p. By comparison, a first moment argument shows that $G(n, p)$ is not k-colorable w.h.p. for $np = 2k \ln(k) - \ln(k)$. Thus, the result determines the threshold for k-colorability up to an additive $2\ln(2)$. This improves upon the result of Achlioptas and Naor (Ann. Math. 2005). Furthermore, we can prove that at the density $(*)$ there occurs a phase transition in the sense that the free entropy is non-analytic. The talk is based on joint work with Dan Vilenchik.