**Rota's conjecture, and positivity of algebraic cycles in the permutohedral toric variety.**
Rota's conjecture predicts that the coefficients of the characteristic polynomial of a matroid form a log-concave sequence. I will outline a proof for representable matroids using the Bergman fan. The same approach to the conjecture in the general case (for possibly non-realizable matroids) leads to several intriguing questions on higher codimension algebraic cycles in the toric variety associated to the permutohedron.