Separating inequalities for nonnegative polynomials that are not sums of squares

Ternary sextics and quaternary quartics are the smallest cases where there exist nonnegative polynomials that are not sums of squares (SOS). A complete classification of the difference between these cones was given by G. Blekherman via analyzing the corresponding dual cones. An exact computation of the extreme rays of these cones in order to separate a fixed nonnegative polynomial that is not a sum of squares from the SOS cone is difficult. We provide a method substantially simplifying this computation for certain classes of polynomials on the boundary of the cone of nonnegative ternary sextics and quaternary quartics. In particular, our method yields separating extreme rays for every nonnegative ternary sextic with at least seven zeros (resp. for every nonnegative quaternary quartic with at least six zeros). At the end, we will discuss some interesting open questions. (Based on joint work with T. de Wolff)