Riemann-Roch Theorems in Discrete Mathematics: An overview

The talk will start with a brief introduction to Riemann-Roch theory on a finite graph intoduced by Baker and Norine, 2007. We will then reformulate the theory in the language of the Laplacian lattice (the lattice generated by the rows of the Laplacian matrix of the graph) and then describe a generalisation of the Riemann-Roch theory to sublattices of the root lattice An. The generalisation is based on studying critical points of a certain simplicial distance function on the lattice. As a consequence, we obtain a geometric proof for the Riemann-Roch theorem for graphs. Furthermore, the geometric proof involves studying a certain simplicial complex, namely the Delaunay triangulation of the Laplacian lattice under the simplicial distance function. We then observe that this Delauanay triangulation is closely related to the notion of Scarf complex. Taking cue from this observation, tools from combinatorial algebra allow us to determine the minimial free resolution of Laplacian lattice ideals that are "generic". The connection between Riemann-Roch theorem for graphs and combinatorial commutative algebra is further clarified by demonstrating that the Riemann-Roch theorem on graphs is essentially the determination of the Alexander dual of a suitably chosen initial ideal of the Laplacian lattice ideal. These considerations also allow us to develop a self-contained Riemann-Roch theory for monomial ideals.

The talk is based on a joint work with Omid Amini, Ecole Normale Superieure, Paris and an ongoing work with Bernd Sturmfels, University of California Berkeley.