Many neighborly polytopes

We say that a d-polytope P is neighborly if every subset of at most $d/2$ vertices is a face of P. We present a new construction that uses Gale duality to build many neighborly polytopes. With it, we can prove that the number of different neighborly d-polytopes with n vertices is at least of order $n^{\lfloor nd/2 \rfloor}$ when $n \geq 2d$, which even improves previously known lower bounds for the number of different combinatorial types of polytopes.