Many neighborly polytopes

We say that a $d$-polytope $P$ is neighborly if every subset of at most $d/2$ vertices is a face of $P$. We present a new construction that uses Gale duality to build many neighborly polytopes. With it, we can prove that the number of different neighborly $d$-polytopes with $n$ vertices is at least of order $n^{(nd/2)}$ when $n \geq 2d$, which even improves previously known lower bounds for the number of different combinatorial types of polytopes.