

### **Relative Upper Bound Theorems.**

The Upper Bound Theorem, conjectured by Motzkin and proved by McMullen, is one of the cornerstones of discrete geometry: Neighborly simplicial polytopes maximize the number of  $k$ -dimensional faces among all  $d$ -dimensional convex polytopes with a fixed number of vertices. Stanley vastly generalized the Upper Bound Theorem by showing that it even holds for general triangulations of spheres (and beyond).

In this talk I will present a generalization of Stanley's approach that yields an algebraic framework for treating relative upper bound problems. As showcases I will present solutions to combinatorial isometric problems and an Upper Bound Theorem for Minkowski sums. The talk will be a scenic tour from geometry to algebra and back.

This is joint work with Karim Adiprasito (IHES).