Rank-1 Bimatrix Games

A bimatrix game is a two-player game in strategic form, one of the basic models of game theory. Its central solution concept is the Nash equilibrium (NE). SSolving", that is, finding one NE of a bimatrix game is an algorithmic problem that has received considerable interest in recent years, and has been shown to be "PPAD-complete", meaning that the path-following argument (such as, for example, the Lemke-Howson algorithm, or Sperner's Lemma) for proving the existence of a NE can encode other, seemingly more difficult fixed-point and economic equilibrium problems.

Kannan and Theobald (2010) introduced a hierarchy of bimatrix games (A, B) via their RANK, defined as the rank of $A + B$, of which the rank-0 or ßero-sum"games are the easiest to solve, via linear programming. They showed that an approximate NE of a fixed-rank game can be found in polynomial time. Recently, Adsul, Garg, Mehta, and Sohoni (STOC 2011) showed that an exact NE of a rank-1 game can be found in polynomial time. Their method describes a path whose intersections with a hyperplane define the NE of the rank-1 game. We explain and streamline their method to show (a) that the path is a well-known "homotopy path", (b) that the NE of a rank-k game can be described as the path of NE of a parameterized game of rank $k-1$ intersected with a hyperplane (unfortunately not with a corresponding polynomial-time algorithm), (c) the construction of a rank-1 game with exponentially many NE (using known exponential paths for parameterized linear programs), answering an open problem by Kannan and Theobald.

The talk will introduce the geometric concepts and path-following methods for finding NE, and conclude with some open problems, for example the complexity of solving rank-2 games.