The lower tail: Poisson approximation revisited

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Abstract

The well-known Janson's inequality gives Poisson-like upper bounds for the lower tail probability $\Pr(X \leq (1-\epsilon)\mathbb{E}X)$ when X is the sum of dependent indicator random variables of a special form. We show that, for large deviations, this inequality is optimal whenever X is approximately Poisson, i.e., when the dependencies are weak. We also present correlation-based approaches that, in certain symmetric applications, yield related conclusions when X is no longer close to Poisson. As an illustration we, e.g., consider subgraph counts in random graphs, and obtain new lower tail estimates, extending earlier work (for the special case $\epsilon = 1$) of Janson, Łuczak and Ruciński.

Joint work with Svante Janson.