Liouville-type theorems for an elliptic system modelling phase-separation and optimal partition problems

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In this talk we consider solutions of the competitive elliptic system

$$\begin{cases} -\Delta u_i = -\sum_{j\neq i} u_i u_j^2 & \text{in } \mathbb{R}^N \\ u_i > 0 & \text{in } \mathbb{R}^N \end{cases} \qquad i = 1, \dots, k, \tag{1}$$

which appears in the analysis of phase separation phenomena for Bose-Einstein condensates with multiple states. We are concerned with the classification of entire solutions, according with their (algebraic) growth rate. The prototype of our main results is the following: for every $d > 0$ there exists $h = h(d, N) \in \mathbb{N}$ such that if $(u_1, \dots, u_k)$ is a solution of (1) and

$$u_1(x) + \cdots + u_k(x) \leq C(1 + |x|^d) \qquad \text{for every } x \in \mathbb{R}^N,$$

then $k \leq h(d, N)$. This means that a bound on the growth of a positive solution imposes a bound on the number of components k of the solution itself. The value $h(d, N)$ is explicitly characterized in terms of an optimal partition problem. We discuss the sharpness of our results and, as a further step, for every $N \geq 2$ we can prove the 1-dimensional symmetry of the solutions of (1) satisfying suitable assumptions, extending known results which are available for $k = 2$. The proofs rest upon a blow-down analysis and on some monotonicity formulae. This is a joint work with Susanna Terracini.