

Colorings of simple hypergraphs and applications

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Abstract

The talk is devoted to the extremal problems concerning colorings of simple hypergraphs. Let $H = (V, E)$ be a simple (i.e. every two edges do not share more than one common vertex) n -uniform hypergraph. We show that if its maximum vertex degree $\Delta(H)$ satisfies the condition

$$\Delta(H) \leq cr^{n-1},$$

where $c > 0$ is some absolute constant, then H is r -colorable. The proof relies on the random recoloring method. As an application of the proof technique we obtain a lower bound for the Van der Waerden number $W(n, r)$:

$$W(n, r) \geq cr^{n-1}.$$