**Finite speed of propagation in a fourth-order degenerate parabolic equation modeling Bose-Einstein Condensation**

In the papers [1] and [2] *continuous weak solutions* of the problem

$$\begin{aligned} u_t &= x^{-\beta}\left(x^\alpha u^{n+2}(u^{-1})_{xx}\right)_{xx}, & x \in \Omega,\, t > 0, \\ x^\alpha u^{n+2}(u^{-1})_{xx} &= \left(x^\alpha u^{n+2}(u^{-1})_{xx}\right)_{xx} = 0, & x = 0,\, t > 0, \\ u_x &= u_{xxx} = 0, & x = L,\, t > 0, \\ u(x,0) &= u_0(x), & x \in \Omega \end{aligned}$$

for a non-empty interval $\Omega = (0, L)$ have been both defined and found. Apart from the existence of such a solution it has been seen

$$\lim_{t \nearrow T_{max}} \|u(\cdot,t)\|_{L^\infty(\Omega)} = \infty$$

for most choices of the initial data $u_0$. Similar to the equation

$$u_t + (u^n u_{xxx})_x$$

which has been discussed in [5], [3] and [4] we are going to consider the time it takes the solution to be positive in an interval $\emptyset \neq \omega \subset u_0^{-1}(\{0\})$. More precisely we seek to prove that in a stark contrast to the case of the *heat equation* we have

$$u(x,t) = 0 \,\text{in}\, (a-\delta, a+\delta) \times (0,T)$$

for positive a, $\delta$ and $T$. For this we derive two *differential inequalities* to first show

$$\{T \in (0,\infty) | \exists \delta > 0 : u(x,t) = 0 \,\text{in}\, (a-\delta, a+\delta) \times (0,T)\} \neq \emptyset$$

and then find an estimate for the positive supremum of this set.

## References


[1] A. Jüngel, M. Winkler. A degenerate fourth-order parabolic equation modeling Bose-Einstein condesation. Part I: Local existence of solutions. *Will be published in: Archive for Rational Mechanics and Analysis*, 2015.

[2] A. Jüngel, M. Winkler. A degenerate fourth-order parabolic equation modeling Bose-Einstein condesation. Part II: Finite-time blow-up. *Will be published in: Communications in Partial Differential Equations*, 2015.

[3] E. Beretta, M. Bertsch, R. Dal Passo. Nonnegative solutions of a fourth-order nonlinear degenerate parabolic equation. *Archive for Rational Mechanics and Analysis 129*, 1995.

[4] F. Bernis. Finite speed of propagation and continuity of the interface for thin viscuous flows. *Advances in Differential Equations, Volume 1, Number 3*, 1996.

[5] F. Bernis, A. Friedman. Higher order nonlinear degenerate parabolic equations. *Journal of Differential Equations 83*, 1990.