On Inversion of Some Integral Transforms in $\mathbb{R}^n$

Rohit Kumar Mishra

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Abstract Integrals of a vector field model measurements in tomographic imaging of moving mediums, in particular, imaging of liquid flow or gas flow, tumor detection, optics and plasma physics. So, inversion of such integrals to reconstruct vector fields are very important.

In this talk, we are going to discuss about the inversion of certain integral transforms. We will start with the recovery of a function from its integrals along lines. Then, we will generalize the similar problems first for vector fields and then for tensor fields.

The ray transform of a symmetric m-tensor field f in $\mathbb{R}^n$ is defined by

$$\mathcal{R}f(x,\xi) = \int_{\mathbb{R}} \langle f(x+t\xi), \xi^m \rangle dt = \int_{\mathbb{R}} f_{i_1 \dots i_m}(x+t\xi)\xi^{i_1} \dots \xi^{i_m} dt \tag{1}$$

where $x \in \mathbb{R}^n$ and $\xi \in \mathbb{S}^{n-1}$ specifies the direction of a line passing through x. For $m = 0$ and $m = 1$ this transform is known as X-ray transform and Doppler transform respectively.

It is well known that only solenoidal part of a symmetric m-tensor field can be recovered from its ray transform (see [1]). In this talk, we will present explicit inversion formula for the reconstruction of solenoidal part of a symmetric m-tensor field from its restricted ray transform. The work of explicit inversion was motivated by the work of Denisjuk [2] who showed that solenoidal part can be recovered and explicit formula for the inversion can be found for $n = 3$ case.

References

[1] V. A. Sharafutdinov. *Integral geometry of tensor fields.* Inverse and Ill-posed Problems Series. VSP, Utrecht, 1994.

[2] Alexander Denisjuk, *Inversion of the x-ray transform for 3D symmetric tensor fields with sources on a curve*, Inverse Problems volume-22, 399–411.