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The convexification effect of Minkowski summation

For a compact subset A of $\mathbb{R}^n$, let $A(k)$ be the Minkowski sum of k copies of A, scaled by $1/k$. It is well known that $A(k)$ approaches the convex hull of A in Hausdorff distance as k goes to infinity. In this talk we will discuss how exactly $A(k)$ approaches the convex hull of A, and more generally, how a Minkowski sum of possibly different compact sets approaches convexity, as measured by various indices of non-convexity.

The non-convexity indices considered will include the Hausdorff distance induced by most general norm, the volume deficit (the difference of volumes), a non-convexity index introduced by Schneider (1975), and the effective standard deviation or inner radius. We will present relationships between those indices and move to discussion of monotonicity of convergence of $A(K)$ with respect to those indices. In particular, we will present a conjecture proposed, a few years ago, by Bobkov, Madiman and Wang, that the volume of $A(k)$ is non-decreasing in k, or in other words, that when the volume deficit between the convex hull of A and $A(k)$ goes to 0, it actually does so monotonically. While this conjecture holds true in dimension 1, we show that it fails in dimension 12 or greater. For other indices of non-convexity, we will present several positive results, including a strong monotonicity of Schneiders index in general dimension, and eventual monotonicity of the Hausdorff distance and effective standard deviation.

Along the way we will demonstrate relations of our results to Combinatorial Discrepancy Theory and Information theory.

The talk is based on a joint work with Mokshay Madiman, Matthieu Fradelizi and Arnaud Marsiglietti.