Bo Berndtsson
Göteborgs Universitet

Symmetrization of plurisubharmonic and convex functions

If u is a real valued function in a domain in $\mathbb{R}^N$, its Schwarz symmetrization, $\hat{u}$, is a radial function, defined in a ball, that is equidistributed with u, so that for any real function

$$\int F(u) = \int F(\hat{u}).$$

The classical Polya-Szegö theorem says that the Newtonian energy, i e the L^2 norm of the gradient, decreases under symmetrization if u vanishes on the boundary of the domain. We will discuss similar results for plurisubharmonic or convex functions, with Monge-Ampere energy instead of Newtonian energy. The main result is - grosso modo - that symmetrization decresases Monge-Ampere energy if the domain we start with is a ball or an ellipsoid, but essentially only then. The proof of the symmetrization inequality uses (complex) Brunn-Minkowski inequalities. The proof of the converse direction uses a curious connection to complex geometry (Kähler-Einstein metrics) in the case of plurisubharmonic functions, and to the theorem of Jörgens-Calabi-Pogorelov in the case of convex functions.

This is joint work with Robert Berman.