

Seminar

Proper Actions on Homogeneous Reductive Spaces

Wintersemester 2017/18

Mo. 14–16 Uhr

- 1. Algebraic groups 1** (23. Oktober) JARO EICHLER
Define algebraic and Lie groups and Lie algebras and give examples (and counterexamples), esp. general linear groups. State some basic facts and define the killing form, solvable Lie algebras, Lie's theorem, nilpotent Lie algebras and Engel's theorem.
Literatur: [Kna02], [Spr98], [Mil13].
- 2. Algebraic groups 2** (6. November) JONATHAN ZACHHUBER
Introduce reductive and semisimple Lie groups/algebras and various criteria [Kna02, §I.7]. Define root systems, Weil chambers [Kna02, Ch. II], i.e. the groundwork for [BHC62].
Literatur: [Kna02], [Spr98], [BHC62], [Mil13].
- 3. Algebraic groups 3** (13. November) MARTIN LÜDTKE
Structure theorem for semi-simple groups, Cartan decomposition, definition of rank, Iwasawa decomposition [Kna02, Ch. VI], parabolic and Borel subgroups [Spr98, Ch. 6], i.e. the groundwork for [BHC62].
Literatur: [Kna02], [Spr98], [BHC62], [Mil13].
- 4. Arithmetic subgroups of algebraic groups** (20. November) NITHI
RUNGTANAPIROM
Quotients of the form G/H (properly discontinuous action vs. GIT [Spr98, Ch. 5.5]), definition of arithmetic lattices and state the results of [BHC62].
Literatur: [Spr98], [BHC62], [Mil13].
- 5. Lattices in $SL(d, \mathbb{R})$ and $SO(p, q)$** (27. November) MARKUS RENNIG
Introduce the Haar measure [Kna02, §IV.2]. Give examples of lattices and show Mahler's criterion (where $G = \mathbb{R}^d$). Show Chevalley's theorem. Construct cocompact lattices and discuss Godement's criterion for $G = SL(d, \mathbb{R})$ and $SO(p, q)$ [Ben14, Ch. 2].
Literatur: [Kna02], [Ben14].

6. **Necessary conditions for compactness of homogeneous spaces** (4. Dezember)
 ROSI MARTIENSSEN
 Discuss [BL92].
Literatur: [BL92].
7. **Proper action on a homogeneous space of reductive type** (11. Dezember) MAX
 BIERI
 Discuss [Kob89], i.e. necessary conditions for the existence of discrete subgroups acting properly discontinuously on a homogeneous space. Section 5 is optional.
Literatur: [Kob89].
8. **Proper action on a homogeneous space of reductive type revisited** (8. Januar)
 MATTEO COSTANTINI
 Discuss necessary and sufficient conditions for the existence of discrete subgroups acting properly discontinuously on a homogeneous space [Ben96, Ch. 3–6].
Literatur: [Ben96].
9. **Schottky groups** (15. Januar) MARTIN MÖLLER
 Introduce Schottky groups by using the Jordan decomposition [Ben96, Ch. 7]. Construct the discrete subgroup Γ [Ben96, §7.4] and deduce the main theorem and its corollaries.
Literatur: [Ben96].

References

- [Ben14] Y. Benoist. “Five lectures on lattices in semisimple Lie groups”. In: *Lecture notes, available on the webpage* (2014).
- [Ben96] Y. Benoist. “Actions propres sur les espaces homogènes réductifs”. In: *Annals of Mathematics* 144 (1996), pp. 315–347.
- [BHC62] A. Borel and Harish-Chandra. “Arithmetic subgroups of algebraic groups”. In: *Annals of Mathematics* 75 (1962), pp. 485–534.
- [BL92] Y. Benoist and F. Labourie. “Sur les espaces homogènes modèles de variétés compactes”. In: *Publications Mathématiques de l’IHES* 76 (1992), pp. 99–109.
- [Kna02] Anthony W. Kna. “Lie Groups Beyond an Introduction”. In: *Birkhäuser Verlag* Second Edition (2002).
- [Kob89] T. Kobayashi. “Proper action on a homogeneous space of reductive type”. In: *Math. Ann.* 285 (1989), pp. 249–263.
- [Mil13] James S. Milne. *Lie Algebras, Algebraic Groups, and Lie Groups*. Available at www.jmilne.org/math/. 2013.
- [Spr98] T. A. Springer. “Linear Algebraic Groups”. In: *Birkhäuser Verlag* Second Edition (1998).