Goethe-Universität Frankfurt
Institut für Mathematik
Sommersemester 2019
June 26, 2019

Seminar
Prof. Dr. Martin Möller

Seminar

Hurwitz numbers and enumerative geometry

Sommersemester 2019

28.-30.Juni

The seminar will take place in Jugendherberge Kloster Leutesdorf, Rheinstraße 25, 56599 Leutesdorf.

1. **Hurwitz numbers and constellations**
(Fr. 14.00-15.00) HANNAH LAUS

Definition of constellations, passports and braid groups ([LZ04, Ch.1,Sec.1.1]), Coverings of the sphere and monodromy action ([LZ04, Ch.1,Sec.1.2]), relation between constellations and covering of the sphere ([LZ04, Ch.1,Sec.1.3]), show that every Riemann surface can be given as a ramified covering of the sphere and vice versa ([LZ04, Ch.1,Prop.1.8.9, Th. 1.8.14]). State the Hurwitz problem.

Literatur: [LZ04, Chapter 1], [RW06], [Cav12] .

2. **Representation theory and application to disconnected Hurwitz numbers**
(Fr. 15.45-16.45) JEONGHOON SO

The main result of the talk is [LZ04, Th. A.1.9], its proof and its interpretation in terms of disconnected Hurwitz numbers [LZ04, Sec. A.1.3]. In order to present it, one needs to recall basic facts about representation theory of finite groups [LZ04, Sec. A.1.1].

Literatur: [LZ04, Appendix A], [DYZ17] .

3. **Moduli functors and moduli space of curves**
(Fr. 17.00-18.00) MATTEO COSTANTINI

Definition of $\mathcal{M}_{g,n}$ as moduli functor and definition of course moduli space [HM98, Sec. 1.A], problem of representability of the functor[HM98, Sec. 2.A] [1], Riemann's computation of dimension of $M_{g,n}$ [Loo99, Sec. 2], definition of Hurwitz moduli functor [HM98, Sec. 1.G].

Literatur: [Zvo12], [Cav12, Sec. 3],[Loo99],[HM98].

[1] The issues of orbifoldiness and stackyness will be taken care during the coffe breaks for interested people.

4. **The Deligne-Mumford compactification $\overline{M}_{g,n}$**
(Sa. 9.30-10.30) VANESSA WALKENHORST

Examples of moduli spaces $M_{0,n}$ and $M_{1,1}$ and compactness problem ([Zvo12, Sec. 1.2]). Definition of stable curves and the Deligne-Mumford compactification $\overline{M}_{g,n}$ with examples ([Zvo12, Sec. 1.4]). Definition of boundary divisors δ's.

Literatur: [Zvo12], [Cav12, Sec. 3].

5. **Special classes of $\overline{M}_{g,n}$**
(Sa. 11.00-12.00) JARO EICHLER

Definition of relative dualizing sheaf for a curve over a base B ([HM98, Ch.3, Sec. A]), fiberwise description of relative dualizing sheaf of the universal curve over $M_{g,n}$ and of its push-forward, i.e. Hodge bundle [HM98, CH.3, Sec. E], definition of ψ-classes ([Zvo12, Sec. 2.1], definition of forgetful maps and attaching maps [Zvo12, Sec. 2.1]

Literatur: [Zvo12], [HM98], [Cav12, Sec. 3],[ACG11].

6. **Low genus examples I: $\overline{M}_{0,n}$**
(Sa. 14.00-15.00) ADRIAN BAUMANN

Explicit construction of $\overline{M}_{0,4}$ and $\overline{M}_{0,5}$ via universal families and blow-ups ([KLP18, §15.4]). Explicit cohomology classes calculation ([Zvo12, Prop.2.3,2.13]) and ψ integrals ([Zvo12, Examples 2.14, 2.15 and 2.16]).

Literatur: [Zvo12], [KLP18] .

7. **Low genus examples II: $\overline{M}_{1,1}$**
(Sa. 15.45-16.45) JOHANNES SCHWAB

Description of $\overline{M}_{1,1}$ and the modular curve ([DS05] or [Hai14]), identification of the Hodge bundle with the canonical bundle of $\overline{M}_{1,1}$, show that modular forms are sections of powers of the Hodge bundle [Zvo12, Sec. 2.2.2]. Computation of first Chern class λ of the Hodge bundle via Gauss-Bonnet and of some ψ integrals (Examples 2.21 and 2.26).

Literatur: [Zvo12], [DS05], [Hai14] .

8. **ELSV formula**
(Sa. 17.00-18.00) MAX BIERI

Recall Hurwitz problem and state the ELSV formula [LZ04, Sec. 5.3.1], explicit formulas for Hurwitz numbers in genus 0 and 1 [LZ04, Sec. 5.3.2].

Literatur: [Eke+01], [LZ04], [Cav12, Sec. 7].

9. **The proof of the Lyashko-Looijenga Theorem in the easy case**
(Sa. 20.00-21.00) DAVID TORRES-TEIGELL

State and prove the easy version of ELSV formula: Lyashko-Looijenga Theorem ([LZ04, Chapter 5, Sec. 5.1.2, 5.1.3])

Literatur: [LZ04, Chapter 5].

10. **Construction of Hurwitz spaces and stable maps**
(So. 9.30-10.30) RICCARDO ZUFFETTI

Recall Hurwitz moduli functor [RW06, Sec. 1.1, 1.2], analytic construction of the Hurwitz course moduli space [RW06, Sec. 1.3], space of stable maps, Lyashko–Looijenga mapping and idea of proof of ELSV formula [Eke+01, Sec.1]

Literatur: [RW06], [Eke+01] .

11. **Completed Hurwitz spaces and proof ELSV formula**
(So. 11.00-12.00) JONATHAN ZACHHUBER

Define cones of principal parts, define completed Hurwitz spaces and proof of ELSV formula ([LZ04, Chapter 5,Sec. 5.3]) .

Literatur: [LZ04, Chapter 5], [Eke+01].

References

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