

## Übungsblatt 6

### Aufgabe 1 (2 Punkte)

Let  $x$  be an *irrational* number with continued fraction expansion  $[a_0, a_1, a_2, \dots]$ . Let  $\frac{p_n}{q_n}$  be the  $n$ -th convergent of  $x$ . Prove that

$$\left| x - \frac{p_n}{q_n} \right| > \frac{1}{q_n(q_n + q_{n+1})}$$

for every  $n \in \mathbb{Z}_{\geq 0}$ .

### Aufgabe 2 (6 Punkte)

Let  $\varepsilon > 0$  and let  $\alpha$  be an *irrational* number. Suppose that the continued fraction expansion  $\alpha = [a_0, a_1, a_2, \dots]$  has infinitely many entries satisfying  $a_{n+1} \geq q_n^\varepsilon$ , where  $q_n$  is the denominator of the  $n$ -th convergent of  $\alpha$ .

- (i) Give an example of irrational number which does not satisfy the condition above.
- (ii) Prove that  $[1, 10^{1!}, 10^{2!}, 10^{3!}, \dots]$  satisfies the previous condition.
- (iii) Prove that if  $\alpha$  satisfies the condition above, then  $\alpha$  is transcendental.

**Hint:** use the Thue–Siegel–Roth Theorem.

**Information:** during the next exercise session, we will have time to discuss a bit about *quadratic fields*. If you want, send an email to [zuffetti@math.uni-frankfurt.de](mailto:zuffetti@math.uni-frankfurt.de) with the question/topic you would like to review. We will briefly recall it together (without video-recording it).

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Please, upload your solutions on the [Olat page](#) of this course, by **14:00** on **Wednesday, 03.06.2020**.