We show that a one-dimensional topological superconductor can be realized in carbon nanotubes, using a relatively small magnetic field. Our analysis relies on the intrinsic curvature-enhanced spin-orbit coupling of the nanotubes, as well as on the orbital effect of a magnetic flux threaded through the nanotube. Tuning experimental parameters, we show that a half-metallic state may be induced in the nanotube. Coupling the system to an Ising superconductor, with an appreciable spin-triplet component, can then drive the nanotube into a topological superconducting phase. The proposed scheme is investigated by means of real-space tight-binding simulations, accompanied by an effective continuum low-energy theory, which allows us to gain some insight on the roles of different terms in the Hamiltonian. We calculate the topological phase diagram and ascertain the existence of localized Majorana zero modes near the edges. Moreover, we find that in the absence of a magnetic field, a regime exists where sufficiently strong interactions drive the system into a time-reversal-invariant topological superconducting phase.